Online Appendices for “Beyond the Diagonal Reference Model: Critiques & New Directions in the Analysis of Mobility Effects”

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Online Appendix A: Prior Contributions Using the DRM

As shown in Figure 1 of the main text, the DRM has been widely and increasingly used to estimate the effects of social mobility on a wide range of outcomes. In this appendix, we provide a schematic summary of the topics addressed in these contributions and the general direction of effects that have been estimated. The tables below highlight the findings from the key studies upon which Figure 1 rests.

Table A.1: Social and Cultural Attitudes

Author	Year	Outcome Measure(s)	Null	+	−	NC
Kulis	1987	Attitudes towards family	X	X		
Marshal and Firth	1999	Life satisfaction	X			
Tolsma et al.	2009	Ethnic attitudes	X	X		
Coulangeon	2013	Musical taste			X	
Daenekindt and Roose	2013	Aesthetic dispositions	X	X		
Daenekindt and Roose	2014	Musical taste		X		
Coulangeon	2015	Musical taste	X	X		
Chan and Turner	2017	Musical taste			X	
Sieben	2017	Child-rearing values	X			
Domański and Karpiński	2018	Culinary taste		X		
Schaeffer	2019	Discrimination perception			X	
Rotengruber and Tyszka	2021	Musical taste		X		
Creighton et al.	2022	Attitudes towards immigration	X	X		
Mijs et al.	2022	Meritocratic beliefs		X		

Notes: Positive effects are given by +, negative effects by −, and zero or near-zero effects by "Null." Those effects that are indeterminate are given as "NC" (Not Classifiable).

Table A.2: Health

Author	Year	Outcome Measure(s)	Null	+	−	NC
Monden and de Graaf	2013	Self-assessed health				X
Missinne et al.	2015	Health care use		X		
Yang	2016	Smoking and drinking			X	
van der Waal et al.	2017	Obesity	X			
Billingsley and Matysiak	2018	Fertility			X	
Dennison	2018	Drug use			X	
Tarrence	2018	Self-rated health		X	X	
Präg and Richards	2019	Stress biomarkers	X			
Steiber	2019	Subjective health satisfaction		X		
Gugushvili et al.	2020	Smoking and drinking		X		
Yang	2020	Smoking	X		X	
Veenstra and Vanzella-Yang	2021	Self-rated health		X		
Zelinska et al.	2021	Self-rated health	X			
Bulczak and Gugushvili	2022	Cardiometabolic risk			X	
Bulczak et al.	2022	Various health measures	X			
Graf et al.	2022	Biological aging		X		
Iveson et al.	2022	Self-rated health	X			
Kempel et al.	2022	Cardiometabolic risk	X			
Luo	2022	Fertility				X
Tarrence	2022	Self-rated health		X		

Notes: Positive effects are given by +, negative effects by −, and zero or near-zero effects by "Null." Those effects that are indeterminate are given as "NC" (Not Classifiable).

Table A.3: Political Outcomes

Author	Year	Outcome Measure(s)	Null	+	−	NC
De Graaf et al.	1990	Left/Right preferences		X		
Weakliem	1992	Left/Right voting		X		
Clifford and Heath	1993	Left/Right voting			X	
Nieuwbeerta and de Graaf	1993	Left/Right voting			X	
Breen and Whelan	1994	Left/Right preferences	X		X	
Graaf et al.	1995	Left/Right preferences			X	
Nieuwbeerta	1995	Left/Right voting		X		
Nieuwbeerta et al.	2000	Left/Right voting	X			
Breen	2001	Left/Right preferences	X		X	
Paterson	2008	Left/Right attitudes				X
Daenekindt et al.	2018	Trust	X	X		
Fan and Yan	2019	Political Participation				X
Jaime-Castillo and Marqués-Perales	2019	Redistribution preferences	X			
Kraus and Daenekindt	2022	Attitudes towards multiculturalism		X		
McNeil	2022	Left/Right attitudes	X	X		
McNeil and Haberstroh	2023	Opinion on Brexit		X		
Wilson et al.	2022	Redistribution preferences			X	

Notes: Positive effects are given by +, negative effects by −, and zero or near-zero effects by "Null." Those effects that are indeterminate are given as "NC" (Not Classifiable).

Table A.4: Psychological Well-being

Author	Year	Outcome Measure(s)	Null	+	−	NC
Houle and Martin	2011	Psychological distress	X			
Zang and de Graaf	2016	Happiness	X			
Daenekindt	2017	Dissociation	X	X		
Schuck and Steiber	2018	Subjective well-being		X		
Gugushvili et al.	2019	Depression		X		
Dhoore et al.	2019	Life satisfaction	X			
Schuck	2019	Subjective well-being	X		X	
Zhao and Li	2019	Subjective well-being		X		
Engzell and Ichou	2020	Subjective social status, perceived financial situation			X	
Gugushvili et al.	2021	Allostatic load		X		
Kaiser and Trinh	2021	Life satisfaction		X		
Zelinska et al.	2021	Subjective well-being		X		
Kwon	2022	Subjective well-being	X			

Notes: Positive effects are given by +, negative effects by −, and zero or near-zero effects by "Null." Those effects that are indeterminate are given as "NC" (Not Classifiable).

Online Appendix B:
Mobility Models & The Identification Challenge: A General Introduction

In this section, we first present a general mobility effects model that incorporates primary parameters capturing the independent effects of origin, destination, and mobility, along with additional cell-specific parameters that represent heterogeneity or interactions among these dimensions. We then introduce a reparameterized version of the conventional model that explicitly distinguishes linear from nonlinear effects. This reparameterization facilitates a deeper understanding of the inherent limitations in identifying mobility effects while also laying the groundwork for the various approaches to analyzing mobility effects we discuss in subsequent sections.

In the conventional mobility effects literature, researchers pursue the identification of unique origin, destination, and mobility effects despite the mathematical dependency among these dimensions. While not explicitly articulated, this pursuit implies a conceptual distinction between observed positions on a mobility table, namely, origin, destination, and mobility, and the underlying causal mechanisms they represent. That is, although destination (D) is mathematically defined as the sum of origin (O) and mobility (M), the bundles of causal mechanisms they proxy for, which we denote as O^*, D^*, and M^*, are theoretically distinct and capable of varying independently. Origin effects (O^*) might represent parental economic resources, cultural socialization, or educational guidance; destination effects (D^*) could capture workplace authority relations, professional network benefits, or class-based consumption opportunities; and mobility effects (M^*) might reflect distinct processes such as status anxiety, reference group changes, or psychological adaptation to class transitions. This distinction supplies the rationale for attempting to identify unique effects of origin, destination, and mobility, notwithstanding their deterministic mathematical relationship. More formally, let O^*, D^*, and M^* denote underlying bundles of causal mechanisms that are allowed to vary freely from each other such that $D^* \neq O^* + M^*$. Mobility effects analysis, as commonly used in the literature, can generally be understood as any analysis using functions of the form $Y = f(O^*, D^*, M^*) + \epsilon$, where ϵ is a normally distributed error term with a mean of zero. However, because O^*, D^*, and M^* are typically unobserved, the observed dimensions of the mobility table, O, D, and M, which have the natural relationship $D = O + M$, are substituted for the underlying causal variables O^*, D^*, and M^* (cf. Bijlsma et al. 2017: 722-724; Clogg 1982; Heckman and Robb 1985). As we show in later sections, it is only under very strong assumptions that one can extract unique "effects" using a conventional analysis of mobility effects.

More specifically, suppose we have data collected on individuals indexed from $r = 1, \ldots, R$, where R is the total number of respondents. Additionally, suppose we have data collected on the underlying causal factors O^*, D^*, M^*, which are coded as categorical variables with levels indexed by $l = 1, \ldots, L$, $p = 1, \ldots, P$, and $n = 1, \ldots, N$, respectively.[1] The mobility effects model can thus be specified as follows:

$$Y = f(O^*, D^*, M^*) + \epsilon = \mu^* + \alpha_l^* + \beta_p^* + \gamma_n^* + \eta_{lpn}^* + \xi_{rlpn}^*, \tag{B.1}$$

where μ^* is the intercept (or overall mean); α_l^*, β_p^*, γ_n^* denote the lth, pth, nth levels of the underlying causal factors for origin, destination, and mobility, respectively; η_{lpn}^* is an additional (orthogonal) term denoting interactions among the underlying factors; and ξ_{rlpn}^* is an individual-level, normally distributed error term with a mean of zero. If one somehow had access to these underlying factors, then, under standard assumptions of consistency and no interference between units, positivity and overlap, and conditional ignorability, we could use Equation B.1 to estimate $\mathbb{E}[Y^{o^*d^*m^*}]$,

[1] For simplicity, and without loss of generality, we will assume that the origin and destination categories are of equal width.

the expected value of the counterfactual outcome if we were to set O^* to some value o^*, D^* to some value d^*, and M^* to some value m^*, each of which, as noted above, are allowed to freely vary. The estimand here is the difference in expected outcomes for two hypothetical individuals who share the same origin o^* and the same destination d^* but differ in their mobility factor m^*. Formally, one can write this "mobility effect" as:

$$\mathbb{E}\big[Y^{(o^*,d^*,m^*)}\big] \; - \; \mathbb{E}\big[Y^{(o^*,d^*,m'^*)}\big],$$

where o^* and d^* are held fixed at the same levels in both expectations, and $m^* \neq m'^*$ represents two different possible mobility "treatments." Under the usual assumptions (consistency, no interference, positivity, ignorability, as well as the assumption that the causal directions are appropriately specified), this difference captures how much the outcome Y would change if origin and destination were fixed to the same values but mobility status were fixed to different values. This is the estimand that the rapidly growing literature on social mobility effects, which overwhelmingly uses the DRM (see Online Appendix A), seeks to identify.

In practice, one typically does not have access to the underlying bundles of causal mechanisms in Equation B.1. Instead of treating them as dimensions of observed positionality on a mobility table, O, D, M, as noted above, are used as proxies for O^*, D^*, and M^*.[2] Specifically, suppose we have a set of categorical variables for $i = 1, \ldots I$ origin groups, $j = 1, \ldots, J$ destination groups, and $k = 1, \ldots, K$ mobility groups, where $k = j - i + I$ and $K = I + J - 1$. The mobility effects model using origin, destination, and mobility as proxies can accordingly be specified using what we call the *Classical Origin-Destination-Mobility (C-ODM) model*:

$$Y = f(O^*, D^*, M^*) + \epsilon \rightarrow f(O, D, M) + \epsilon = \mu + \alpha_i + \beta_j + \gamma_k + \eta_{ijk} + \xi_{rijk}, \qquad \text{(B.2)}$$

where μ is the intercept (or overall mean); α_i, β_j, γ_k denote the ith, jth, kth observed levels of origin, destination, and mobility, respectively; η_{ijk} is an additional (orthogonal) term denoting interactions; and ξ_{rijk} is an individual-level, normally-distributed error term with a mean of zero. To reiterate, Equation B.2 is based on the implicit assumption that O, D, and M (and their respective indices) can be treated as surrogates for O^*, D^*, and M^* (and their respective indices). To simplify the exposition, we will accordingly refer to α_i, β_j, and γ_k as the "true" origin, destination, and mobility effects, but the reader should keep in mind that this is shorthand for referring to α^*_l, β^*_p, γ^*_n (for a similar point, see Fosse and Winship 2019a).

Unfortunately, the basic mobility effects model outlined in Equation B.2 suffers from a fundamental identification problem that goes beyond the identification problem common to all linear models using categorical variables as inputs.[3] This problem was vividly illustrated by the sociologist

[2] An alternative strategy is to shift away from modeling general "effects" and instead focus on examining the effects of specific variables that are thought to capture particular origin-, destination-, or mobility-related processes. For example, rather than attempting to model an omnibus "mobility effect" on, say, voting behavior or political preferences (e.g., Clifford and Heath 1993; De Graaf et al. 1995), one might examine how specific mobility-related events, such as a spell of unemployment or changes in job tasks, affect the likelihood of voting for a particular political party (e.g., Turner and Ryan 2023; Wiertz and Rodon 2021). However, because this approach focuses on understanding the effects of particular mechanisms rather than global origin, destination, and mobility effects, it may be seen as a shift away from mobility effects analysis as it has been traditionally understood in the literature.

[3] The common identification problem is that, with an intercept in the model, there is one more level than can be estimated for the origin, destination, and mobility effect. Although common, interpretation errors can ensue: For example, in a related literature on APC models, it has been shown that for some estimators seemingly trivial changes in coding schemes, such as the level used as the reference category, can generate dramatically different results (Fosse and Winship 2018). In the discussion that follows, we will assume that sum-to-zero constraints are applied, such that

Hubert Blalock (1966: 53), who posed the following thought experiment: "Suppose an unscrupulous demon were to perform certain legitimate mathematical manipulations, presenting to us some new equations with different numerical values for the slopes. Could we ever discover the hoax?" Unfortunately, with respect to the analysis of mobility effects, the answer is in the negative: the linear effects are not identified in conventional mobility models, and estimates are compatible with an infinite range of possible values (Blalock 1966).[4] Intuitively, this is simply because there is not enough information to identify all three linear effects from the data alone (for a related discussion and proofs, see Fosse and Winship 2018).

It is worth emphasizing that the identification challenge in mobility research shares important similarities with the classic age-period-cohort (APC) problem, in which $\text{Age} + \text{Cohort} = \text{Period}$ creates a linear dependency that cannot be resolved with data alone. However, APC analyses typically rely on a Lexis table indexing temporally-based dimensions, namely, age, historical period, and birth cohort, which serve as proxies for life-cycle, generational, and period-based causal processes (e.g., see Fosse and Winship 2019b). By contrast, mobility research is grounded in structural dimensions, namely, class origin, class destination, and social mobility, that proxy class- and movement-based causal processes. These different substantive applications have informed the history of model development in both domains. Despite these distinct substantive interpretations, however, both APC and origin-destination-mobility models confront the same underidentification challenge: neither set of "effects" can be uniquely disentangled without additional, often strong, assumptions.

An alternative formulation of the C-ODM model (see Equation B.2) helps clarify the nature of the identification problem. By orthogonalizing the linear from the nonlinear terms, we can specify what we call the *Linearized Origin-Destination-Mobility (L-ODM) model*:

$$\mu_{rijk} = \mu + \alpha(i - i^*) + \beta(j - j^*) + \gamma(k - k^*) + \widetilde{\alpha}_i + \widetilde{\beta}_j + \widetilde{\gamma}_k + \eta_{ijk} + \xi_{rijk} \qquad \text{(B.3)}$$

where the asterisks denote midpoint or referent indices $i^* = (I + 1)/2$, $j^* = (J + 1)/2$, and $k^* = (K + 1)/2$; α, β, and γ denote the linear effects of origin, destination, and mobility, respectively; and $\widetilde{\alpha}$, $\widetilde{\beta}$, and $\widetilde{\gamma}$ represent the origin, destination, and mobility nonlinear effects, respectively; η_{ijk} is, as before, an additional (orthogonal) term denoting interactions; and ξ_{rijk} is a normally-distributed individual-level error term with a mean of zero. To identify the levels of the parameters given the inclusion of the intercept, sum-to-zero constraints are applied to the linear and nonlinear parameters.

The C-ODM and L-ODM models are equivalent representations of class data grouped by origin, destination, and mobility in the sense that a model fitted using either specification will result in the same predicted values of the outcome.[5] However, the L-ODM model has a significant advantage over the C-ODM model. Due to the linear dependence among origin, destination, and mobility, as well as the fact that origin, destination, and mobility parameters combine slopes with deviations, even after applying sum-to-zero constraints, in general no parameters are identified in the C-ODM model other than the overall mean (cf. Fosse and Winship 2018).[6] By contrast, after applying the

$\sum_{i=1}^{I} \alpha_i = \sum_{j=1}^{J} \beta_j = \sum_{k=1}^{K} \gamma_k = 0$, with the last category of the origin, destination, and mobility variables dropped.

[4] This does not mean, however, that, based on substantive or theoretical knowledge, one would necessarily believe that all values are equally valid (e.g., see Fosse and Winship 2019b).

[5] More technically and precisely, the two models are equivalent in that they span the same linear subspace of the data.

[6] However, if I, J, and/or K are odd, then under conventional "normalization" assumptions the corresponding mean parameters $\alpha_{(I+1)/2}$, $\beta_{(J+1)/2}$, and/or $\gamma_{(K+1)/2}$ will be identified (see Smith 2021 for a similar point).

sum-to-zero constraints, only the three linear effects in the L-ODM model remain unidentified.[7] This greatly simplifies the nature of the identification problem and, as we show later, allows one to use graphical tools for visualizing and partially identifying the parameters of a mobility effects model. Moreover, as we demonstrate in the next section, the L-ODM can be used to clarify the assumptions underlying the current wave of studies on mobility effects.

[7] We elaborate on this property later when we discuss the bounding approach to mobility effects models.

Online Appendix C: Further Simulations of the Bias in the DRM

Here we present results from simulations with various values of the nonlinear effects for origin and destination. Note that in these simulations we assume that the proportionality constraint holds for the nonlinear origin and destination effects, but not necessarily for their respective linear effects. In all simulations we fix the true linear effects at the same magnitude and sign, namely, $\alpha = \beta = \gamma = 0.250$. We present the nonlinear effects in terms of quadratic (second-order) components, but similar results could be obtained using alternative representations of the nonlinearities (e.g., deviations from the linear effects). To clarify the sensitivity of the DRM to the nonlinear effects, in all simulations we keep the true origin, destination, and mobility linear effects fixed at 0.250. The top half of Table C.1 displays the results across different values of the quadratic origin parameter, while the bottom shows the results across different values of the quadratic destination parameter. As with the previous tables of simulations, the shaded rows denote those specific simulations in which the data generating parameters happen to be recovered by the DRM.

Table C.1: Sensitivity of Estimated Mobility Linear Effect
to Values of True Origin and Destination Nonlinear Effects

	True DGP			DRM Estimates			Bias ($\widehat{\gamma} - \gamma$)				
	α^2	β^2	γ	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\gamma}$	$(\beta)(\widehat{w}_o)$	$-$	$(\alpha)(\widehat{w}_d)$	$=$	$\widehat{\gamma} - \gamma$
Varying α^2:	0.000	0.050	0.250	0.000	0.500	0.000	$(0.250)(\frac{0.000}{0.050})$	$-$	$(0.250)(\frac{0.050}{0.050})$	$=$	-0.250
	0.050	0.050	0.250	0.250	0.250	0.250	$(0.250)(\frac{0.050}{0.100})$	$-$	$(0.250)(\frac{0.050}{0.100})$	$=$	0.000
	0.100	0.050	0.250	0.333	0.167	0.333	$(0.250)(\frac{0.100}{0.150})$	$-$	$(0.250)(\frac{0.050}{0.150})$	$=$	0.083
	0.250	0.050	0.250	0.417	0.083	0.417	$(0.250)(\frac{0.250}{0.300})$	$-$	$(0.250)(\frac{0.050}{0.300})$	$=$	0.167
	0.500	0.050	0.250	0.455	0.046	0.455	$(0.250)(\frac{0.500}{0.550})$	$-$	$(0.250)(\frac{0.050}{0.550})$	$=$	0.205
	1.000	0.050	0.250	0.476	0.024	0.476	$(0.250)(\frac{1.000}{1.050})$	$-$	$(0.250)(\frac{0.050}{1.050})$	$=$	0.226
Varying β^2:	0.050	0.000	0.250	0.500	0.000	0.500	$(0.250)(\frac{0.050}{0.050})$	$-$	$(0.250)(\frac{0.000}{0.050})$	$=$	0.250
	0.050	0.050	0.250	0.250	0.250	0.250	$(0.250)(\frac{0.050}{0.100})$	$-$	$(0.250)(\frac{0.050}{0.100})$	$=$	0.000
	0.050	0.100	0.250	0.167	0.333	0.167	$(0.250)(\frac{0.050}{0.150})$	$-$	$(0.250)(\frac{0.100}{0.150})$	$=$	-0.083
	0.050	0.250	0.250	0.083	0.417	0.083	$(0.250)(\frac{0.050}{0.300})$	$-$	$(0.250)(\frac{0.250}{0.300})$	$=$	-0.167
	0.050	0.500	0.250	0.046	0.455	0.046	$(0.250)(\frac{0.050}{0.550})$	$-$	$(0.250)(\frac{0.500}{0.550})$	$=$	-0.205
	0.050	1.000	0.250	0.024	0.476	0.024	$(0.250)(\frac{0.050}{1.050})$	$-$	$(0.250)(\frac{1.000}{1.050})$	$=$	-0.226

Notes: Number of origin and destination groups is set at $I = 5$ and $J = 5$, respectively, for all simulations. Sample size for each simulation is $R = 5,958$. For all simulations the true linear effects are fixed at $\alpha = \beta = \gamma = 0.250$ and the true nonlinear effects other than α^2 and β^2 are fixed at zero. For simplicity, and without loss of generality, we assume no random error. Shaded rows denote simulations in which the DRM recovers the true mobility linear effect ($\widehat{\gamma} = \gamma$). The bias arises due to the fact that the underlying origin and destination linear effects do not obey the proportionality constraints of the estimated weights, which are a function of the underlying nonlinear effects. For example, in the top row the estimated weights for origin and destination are $\widehat{w}_o = 0.000$ and $\widehat{w}_d = 1.000$, respectively. Yet the actual weights needed to recover the true origin and destination effects are $w_o = 0.500$ and $w_d = 0.500$, respectively.

Online Appendix D: Additional Complications for Point-Identified or Partially-Identified Mobility Effects

So far little has been stated about the meaning of a mobility "effect." In this appendix we outline the conceptualization of a mobility effect implied by the conventional mobility effects literature, outlining a number of challenges.

Graphical Causal Models

Figure D.1 presents the basic setup of a mobility effects model in terms of directed acyclic graphs (DAGs). As is common in the sociological literature, we treat these DAGs as the graphical embodiment of Pearl's (2009) nonparametric structural equation models (NPSEMs), using them to explicitly encode the underlying causal structure among the relevant variables. Panel (a) shows the graphical model for the causal effects of the origin (O^*), destination (D^*), and mobility (M^*) underlying factors on an outcome (Y) along with a background variable (X).[8] Filled circles denote observed variables while hollow circles denote unobserved variables. Note that, because the causal factors are unobserved, they are denoted with hollow circles. As well, for simplicity of presentation we have omitted idiosyncratic causes that affect the three underlying, unobserved factors (U_{O^*}, U_{D^*}, U_{M^*}) and the outcome (U_Y).

In a conventional mobility effects analysis, the observed variables O, D, and M, which have the natural relationship $D := O + M$ (where $:=$ means "is defined as"), are substituted for the underlying causal variables O^*, D^*, and M^*.[9] This scenario is shown in panel (b) of Figure D.1. The double lines indicate the linear dependency among the dimensions. Note that these variables, unlike O^*, D^*, and M^*, are observed, as indicated by the solid points. As well, the observed variables O, D, and M are not affected by idiosyncratic causes, as they are deterministically related dimensions of the mobility table.

As we have discussed in previous sections, the scenario in Figure D.1(b) is problematic because of the linear dependence among origin, destination, and mobility. Even assuming that the underlying, unobserved factors are additive, with minimal interactions,[10] there is still not enough information to uniquely estimate all three causal effects. In operational terms, this means that we can only condition on two of the three variables (or, more precisely, their linear components). This is clearly a problem, as the graphical model in Figure D.1 shows that estimating just two of the three causal effects will lead to biased estimates.

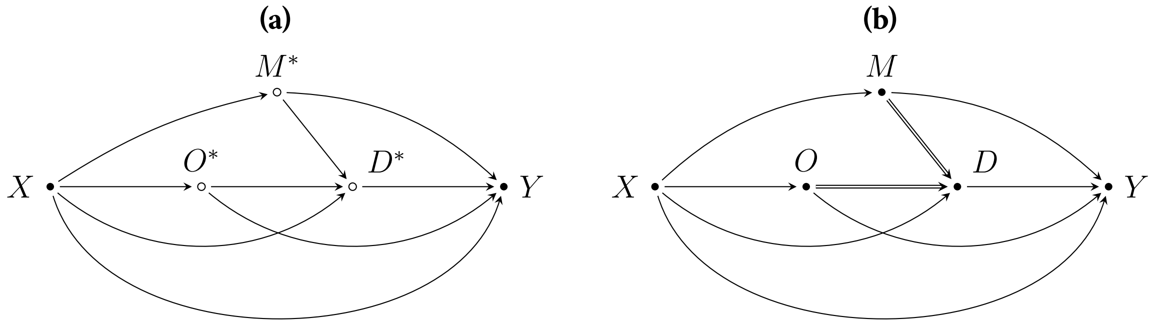
For the sake of the present discussion, however, let us assume that we have somehow obtained the underlying causal factors O^*, D^*, and M^*. Suppose further that we are somehow able to obtain estimates that correspond to causal graph in Figure D.1(a). Even in such an idealized scenario, serious problems remain in interpreting the estimates of these effects. We outline four such problems below. It is important to understand that these issues arise in all models of mobility effects, and add a further layer of complexity to identification and estimation.

[8] As outlined previously, in conventional models of mobility effects, origin, destination, and mobility are implicitly treated as surrogates for unobserved factors that actually generate an outcome of interest. Again, let O^*, D^*, and M^* denote underlying causal factors that are allowed to freely vary from each other such that D^* need not equal $O^* + M^*$.

[9] More specifically, for a mobility table with $i = 1, \ldots, I$ origin groups, $j = 1, \ldots, J$ destination groups, and $k = 1, \ldots, K$ mobility groups, it is the case that $j = i + k - I$ and $K = I + J - 1$.

[10] It is important to note that saying that O^*, D^*, and M^* are additive is not the same as saying that O, D, and M are additive. The reason is that the unobserved factors lie on a three-dimensional tensor, while the observed dimensions lie on a two-dimensional mobility table. In the two-dimensional mobility table, the nonlinearities in any one of the dimensions will appear "interactive" with respect to the other two dimensions. For example, the class destination nonlinearities will appear at different mobility levels for different class origin groups.

Figure D.1: Graphical Models of Origin, Destination, and Mobility Effects



Notes: Panel **(a)** shows the graphical model for the causal effects of the origin (O^*), destination (D^*), and mobility (M^*) underlying, unobserved factors on an outcome (Y) along with a background variable (X). Filled circles denote observed variables while hollow circles denote unobserved variables. Idiosyncratic causes that affect the three underlying factors (U_{O^*}, U_{D^*}, U_{M^*}) and the outcome (U_Y) are omitted for simplicity of presentation. Panel **(b)** shows the graphical model with the observed origin (O), destination (D), and mobility (M) dimensions used as proxies for the underlying, unobserved causal factors. Double lines indicate the linear dependency among the dimensions. Here O, D, and M are deterministic indices, so we omit idiosyncratic causes for them in this schematic.

Parallel-World Counterfactuals

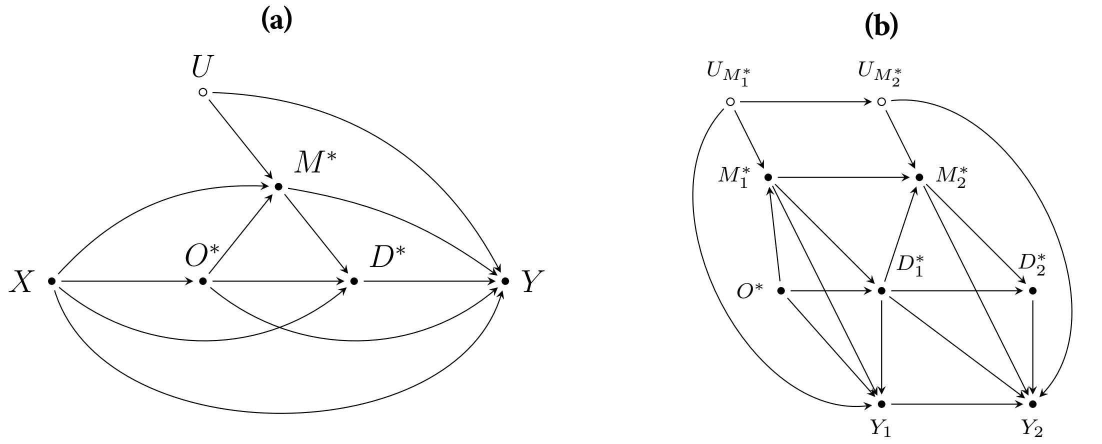
In the context of mobility effects models, counterfactuals can be defined in terms of intervening to fix (or set) some values of underlying origin, destination, and mobility factors for an individual. We use the counterfactual notation Y^{o^*,d^*,m^*}, where superscripts index hypothetical interventions on the causal factors O^* (origin), D^* (destination), and M^* (mobility). We could thus define, for a particular individual, a counterfactual outcome as $Y^{o^*=\text{low},d^*=\text{high},m^*=\text{down}}$, where low, high, and down refer to bundles of causal processes. In this case, the individual is fixed to values of the underlying mechanisms that imply upward mobility, while also being fixed to mechanisms that imply downward mobility. This is not intrinsically problematic, as various estimated causal effects, such as natural direct and indirect effects, invoke parallel-world counterfactuals. It does mean, however, that there is no obvious way for a real-world intervention, such as a randomized experiment, to generate the expected value of this counterfactual for a given population.[11]

The Consistency Assumption

Suppose we believe that we have identified a causal effect for mobility. To endow this estimate with a formal counterfactual interpretation, one must invoke the assumption of consistency (Hernan 2016). Formally, given an individual respondent r is exposed to $M_r = m$, the counterfactual outcome Y_r^m is said to be "consistent" with the observed outcome Y_r if $Y_r^m = Y_r$. This assumption is violated if there are multiple ways to obtain a given level of exposure, thereby generating different counterfactuals. This assumption is particularly likely to be violated for composite variables that reflect multiple underlying features of the data. For example, the causal effect of cholesterol on health is "inconsistent" because the effect on health is very different depending on whether or not one intervenes to raise cholesterol by increasing HDL ("good" cholesterol) versus LDL ("bad" cholesterol). The practical advice is to use variables that correspond to narrowly defined exposures,

[11] Furthermore, as with the literature on natural direct and indirect effects, which has drifted toward so-called "interventional analogues" of mediator effects (Vansteelandt and Daniel 2017), it is unclear whether such parallel-world counterfactuals are actually of interest to applied researchers.

Figure D.2: Social Mobility as a Confounder



Notes: Panel (a) shows the underlying graphical model, which now includes a path between O^* and M^* as well as an additional unobserved causal variable U. It is assumed that the underlying causal variables O^*, D^*, and M^* are all observed. Because M^* lies on the $O^* \rightarrow Y$ path and, with respect to O^* and the unobserved variable U, is a collider, conditioning on M^* using conventional techniques biases estimation by (1) blocking part of the $O^* \rightarrow Y$ effect and (2) opening the backdoor path $O^* \rightarrow M^* \leftarrow U \rightarrow Y$. Panel (b) shows the graphical model if there are multiple destination and mobility causal factors measured at subsequent time periods (D_1^*, M_1^*, D_2^*, M_2^*), with multiple outcomes (Y_1, Y_2) and multiple unobserved confounders ($U_{M_1^*}$, $U_{M_2^*}$). Similar issues arise in this more complicated setting.

with correspondingly well-defined counterfactuals.

Note, however, that this is a subjective decision that depends on one's expertise and tolerance for ambiguity. For example, Rehkopf et al. (2016) outline ways in which neighborhoods, income, and education can each be understood to violate the consistency assumption, despite widespread agreement in sociology and related fields that these are all causal variables. They consider education as a variable that violates the consistency assumption, inasmuch as it has an effect on the outcome via, for example, "improvements in knowledge and cognitive skills, credentials that are valued on the labor market, status improvements, and changes to the individual's social network (2016: 66)." By their logic, measures of social class and class mobility are clearly composite variables that reflect a variety of underlying mechanisms (e.g., Wright 2005) and thus lead to ill-defined, ambiguous counterfactuals. This suggests that, because O^*, D^*, and M^* bundle heterogeneous mechanisms, the basic mobility effects approach should be replaced by a more targeted approach that focuses on more specific, well-defined causal mechanisms.

Unobserved Confounding

To identify the causal effect of social mobility, we must assume that mobility is not confounded with the outcome. This implies conditioning on relevant background variables and avoiding conditioning on post-exposure confounders, which would block some of the effects of mobility. However, the assumption of no unobserved confounding is particularly thorny with respect to mobility effects models. This is closely related to the consistency assumption. Because these are omnibus factors representing multiple causal mechanisms and background variables, it is difficult to imagine which unobserved variables would confound the effect of mobility on a given outcome. Again, more narrowly defined exposures are helpful in figuring out what variables are potential confounders, but this will lead the researcher away from the analysis of mobility effects as they have been conventionally understood in the literature.

Mobility as a Confounder

Finally, a fourth problem with models of mobility effects concerns the fact that social mobility itself is a confounding variable. A careful inspection of Figure D.1 reveals that we have assumed that the underlying origin variable has no causal effect on the underlying mobility variable. This is unrealistic in practice. Presumably, the bundle of causal factors for class origin affects not only those for class destination, but also mobility. This scenario is illustrated in Figure D.2(a), which now includes a path between O^* and M^* as well as an additional unobserved causal variable U. To reiterate, following the assumptions of our present discussion, it is assumed that the underlying causal variables O^*, D^*, and M^* are all observed.

Suppose we want to identify the total causal effect of O^* on Y. Assuming we have observed the underlying variables in Figure D.2(a), conventional, naive adjustment techniques would introduce potential bias. Specifically, because M^* lies on the $O^* \rightarrow Y$ path and, with respect to O^* and the unobserved variable U, is a collider, conditioning on M^* using conventional techniques biases estimation by (1) blocking part of the $O^* \rightarrow Y$ effect and (2) opening the backdoor path $O^* \rightarrow M^* \leftarrow U \rightarrow Y$. The same issues extend to more complicated settings, such as that shown in Figure D.2(b), with multiple class destination and mobility variables, as well as multiple confounders.[12]

[12]Assuming one has observed factors for origin, destination, and mobility, then various methods for time-varying confounding could be used to estimate the effects, such as structural nested models (Vansteelandt and Joffe 2014), marginal structural models (Robins et al. 2000), or residualized regression models (Wodtke and Xiang 2020).

Online Appendix E: Summarizing Variability on the Mobility Table

In this appendix we outline models for describing structural and dynamic inequalities in a mobility table. We first present three different models for summarizing variation in a mobility table based on re-indexing the L-ODM model by origin-destination, destination-mobility, and origin-mobility, respectively. We then show, using the logic of omitted variable bias and matrix algebra, how these models clarify what is actually estimated when fitting all three possible one-factor models and all three possible two-factor models. These results show that, in general, that for describing patterns of mobility either a model of the form $Y = f(D)$ or $Y = f(O, M)$ is to be preferred. Throughout this appendix we let $i = 1, \ldots, I$ index the origin groups, $j = 1, \ldots, J$ the destination groups, and $k = 1, \ldots, K$ the mobility groups, where $k = j - i + I$ and $K = I + J - 1$. As well, we let $r = 1, \ldots, R$ index the respondents (i.e., individuals) in the data set.

1. Three Models for Describing Dynamic and Structural Inequality

As noted in the main text, instead of attempting to identify unique or "pure" effects, one can use the L-ODM model to identify structural and dynamic processes operating on a mobility table. The key insight is that we can project the three-dimensional (unidentified) L-ODM model onto a two-dimensional surface (i.e., a mobility table) by exploiting the fact that mobility, origin, and destination are linearly related. Because there are three different ways to index a mobility table (i.e., origin-mobility, origin-destination, destination-mobility), there are three distinctly different models for describing patterns on a mobility table.[13] We outline each of these models below. Although each model is indexed by two dimensions, because each model contains parameters for all three dimensions, we will refer to them as "three-factor" models.

The first model is based on taking the L-ODM model and re-specifying it as an origin-mobility model. Note that, given an origin-destination mobility table, $j = i + k - I$ and $J = K - I + 1$. Substituting for j and J in the L-ODM model and rearranging terms leads to what we call the *Structural and Dynamic Inequality model* or the *SDI model* for short:

$$\mu_{rijk} = f(O, M) = \mu + \Gamma_1(i - i^*) + \Gamma_2(k - k^*) + \widetilde{\alpha}_i + \widetilde{\beta}_{[i+k-I]} + \widetilde{\gamma}_k + \eta_{i[i+k-I]k} + \xi_{ri[i+k-I]k}, \quad \text{(E.1)}$$

where $\Gamma_1 = \alpha + \beta$ and $\Gamma_2 = \gamma + \beta$, or the social structure (ST) slope and the social mobility (SM) slope, respectively. As a result of the substitution of the sum of the origin and mobility indices for the destination indices (that is, $j = i + k - I$ and $J = K - I + 1$), the outcome is simply a function of origin, indexed by i, with corresponding parameters representing structural inequalities, and mobility, indexed by k, with corresponding parameters representing dynamic inequalities. This model is identified (i.e., the design matrix is of full rank) as it does not contain a separate linear term for destination, which is instead combined with the origin and mobility linear terms, respectively.

The second model is based on expressing the parameters of the L-ODM model in terms of origin and destination. Note that, given an origin-destination mobility table, $k = j - i + I$ and $K = I + J - 1$. Substituting for k and K in the L-ODM model and rearranging terms results in what we call the *Intra-Destination Differences and Structural Inequality model* or, for short, the *Diff-SI model*:

$$\mu_{rijk} = f(O, D) = \mu + (\Gamma_1 - \Gamma_2)(i - i^*) + \Gamma_2(j - j^*) + \widetilde{\alpha}_i + \widetilde{\beta}_j + \widetilde{\gamma}_{j-i+I} + \eta_{ij[j-i+I]} + \xi_{rij[j-i+I]}, \quad \text{(E.2)}$$

where $\Gamma_1 - \Gamma_2 = (\alpha + \beta) - (\gamma + \beta) = \alpha - \gamma$ and $\Gamma_2 = \gamma + \beta$. The difference $\Gamma_1 - \Gamma_2$ in Equation E.2 is a slope of differences within the class destination, while Γ_2 is simply the total realized mobility slope from the SDI model, but indexed by destination ($j = 1, \ldots, J$) instead of

[13] Note that these models take the general form of $Y = f(O, D)$, $Y = f(D, M)$, and $Y = f(O, M)$.

by mobility levels ($k = 1, \ldots, K$). Similar to the SDI model, the Diff-SI model is identified because it does not contain a unique linear term for social mobility, which is instead absorbed into the origin and destination linear terms.

Finally, the third logically possible model entails expressing the parameters of the L-ODM model in terms of destination and mobility. Note that, given an origin-destination mobility table, $i = j - k + I$ and $I = K - J + 1$. Substituting for i and I in the L-ODM model and rearranging terms results in what we call the *Dynamic Inequality and Intra-Destination Differences model* or, for short, the *DI-Diff model*:

$$\mu_{rijk} = f(D, M) = \mu + \Gamma_1(j - j^*) + (\Gamma_2 - \Gamma_1)(k - k^*) + \widetilde{\alpha}_{j-k+(K-J+1)} + \widetilde{\beta}_j + \widetilde{\gamma}_k + \eta_{[j-k+(K-J+1)]jk} + \xi_{r[j-k+(K-J+1)]jk}, \tag{E.3}$$

where $\Gamma_2 - \Gamma_1 = (\gamma + \beta) - (\alpha + \beta) = \gamma - \alpha$ and $\Gamma_1 = \alpha + \beta$. The difference $\Gamma_2 - \Gamma_1$ in Equation E.3 is an overall slope of differences within destination classes, while Γ_1 is simply the ST slope from the SDI model, but indexed by class destination ($j = 1, \ldots, J$) instead of class origin ($i = 1, \ldots, I$). Similar to the previous two models, the DI-Diff model is identified because it does not include a separate linear term for origin, which is instead absorbed into the destination and mobility linear terms.

All three models outlined above provide the same estimates of the intercept and the origin, destination, and mobility nonlinearities. However, unlike the SDI model, the slopes indexed by origin and mobility in Equations E.2 and E.3, respectively, are what can be deemed "synthetic," conflating structural with dynamic inequalities. This is because these slopes are estimated while conditioning on the class destination linear component, and, as such, represent heterogeneous origin-mobility comparisons within a given class destination. In fact, it is only under very specific circumstances that Equations E.2 and E.3 will give unbiased estimates of structural and dynamic inequalities, respectively. Specifically, the Diff-SI model will produce the correct estimate of Γ_1 only if Γ_2 happens to be zero, while the DI-Diff model will give the correct estimate of Γ_2 only if Γ_1 happens to be zero. That is, $\Gamma_1 - \Gamma_2 = \Gamma_1$ only if $\Gamma_2 = 0$, and $\Gamma_2 - \Gamma_1 = \Gamma_2$ only if $\Gamma_1 = 0$. Thus, for the purposes directly estimating structural and dynamic inequalities, the SDI model is strongly preferred over the Diff-SI and DI-Diff models.

In the following sections, we examine the properties of all six logically possible one-factor and two-factor class models for a given mobility table or, equivalently, data with class origin, destination, and mobility variables. These models are listed in Table E.1. For each one- or two-factor model, we use a corresponding three-factor model to clarify exactly what is being estimated. For example, as shown in the first row of Table E.1, to understand the properties of the marginal destination model, which is a one-factor model, we use the SDI model. Similarly, to clarify the estimates of the two-factor origin-destination model (i.e., Duncan's "square additive model"), we use the DI-Diff model. Note that we consider all of these models to be descriptive, such that these various models are different ways of summarizing aggregate-level variability on a mobility table without relying on information purely external to the data.

Table E.1: Comparison of Class Models for a Mobility Table

Reference Model	One-Factor Model	Two-Factor Model
SDI model (E.1)	marginal destination (E.6)	origin-mobility (E.14)
Diff-SI model (E.2)	marginal mobility (E.8)	origin-destination (E.10)
DI-Diff model (E.3)	marginal origin (E.4)	destination-mobility (E.12)

Notes: This table outlines the various one- and two-factor models analyzed based on a corresponding reference model using matrix algebra and the logic of omitted variable bias. For example, the parameters of the marginal destination model and origin-mobility model are interpreted using the SDI model. Equation numbers are in parentheses. Note that these models are all treated as descriptive, and are thus distinct from a conventional mobility effects model, which has the form of $f(O^*, D^*, M^*)$, where O^*, D^*, and M^* are unobserved causal factors proxied by O, D, and M.

2. Interpreting the Parameters of One-Factor Class Models

In this section, we outline the three logically possible one-factor models (based on either origin, destination, or mobility) that can be used to describe the main patterns on a mobility table. For each one-factor model, we outline the relationship between the model's parameters and those from a corresponding model that includes all three factors (see Equations E.1, E.2, and E.3). To avoid confusion with corresponding terms in the three-factor models outlined previously, we use asterisks to denote the parameters in the one-factor models. In general, among all three one-factor models, we recommend using only the marginal destination model, as the underlying slope estimated by this model can be straightforwardly interpreted as a weighted sum of the ST and SM slopes from the SDI model.

i. Marginal Origin Model

The first logically possible one-factor model is the origin class mobility model, which has the following form:

$$\mu_{rijk} = f(O) = \mu^* + \alpha_i^* + \epsilon_{ri}^*, \tag{E.4}$$

where μ^* is the intercept; α_j^* are parameters for class origin using sum-to-zero deviation (or "effect") coding; and ϵ_{ri}^* denotes individual-level error. Using the DI-Diff model as a reference (see Equation E.3), the class origin model outlined in Equation E.4 can be shown to be equivalent to the following:

$$\begin{aligned} \mu_{rijk} &= \underbrace{(\mu + \phi_\mu)}_{\mu^*} + \underbrace{\big(\alpha_M + \phi_{\alpha_M}\big)(i - i^*) + (\widetilde{\alpha}_i + \phi_{\widetilde{\alpha}_i})}_{\alpha_i^*} + \underbrace{(\epsilon_{rijk} + \eta_{ijk} + \nu_{ijk})}_{\epsilon_{ri}^*} \quad \text{and} \\ \alpha_M &= \big(\Gamma_1\omega_{(j,i)} + (\Gamma_2 - \Gamma_1)\omega_{(k,i)}\big), \end{aligned} \tag{E.5}$$

where μ is the intercept; Γ_1 is the ST slope; $\Gamma_2 - \Gamma_1 = \gamma - \alpha$ is the intra-destination slope; α_M is the marginal origin slope; $\omega_{(j,i)}$ is the relationship between the destination linear component and the origin linear component conditional on the intercept and origin nonlinear components; $\omega_{(k,i)}$ is the relationship between the mobility linear component and the origin linear component conditional on the intercept and origin nonlinear components;[14] $\widetilde{\alpha}_i$ is the ith origin nonlinearity; ϕ_μ, ϕ_{α_M} and $\phi_{\widetilde{\alpha}_i}$ are bias terms for the intercept, marginal origin slope, and the ith origin nonlinearity; ϵ_{rijk} is individual-level error; η_{ijk} denotes unique cell-specific heterogeneity; ν_{ijk} denotes additional heterogeneity attributable to class destination and mobility. The terms in brackets below Equation

[14] Those in the upper class can only be downwardly mobile or stay the same, while those in the lower class can only be upwardly mobile or stay the same. Accordingly, the relationship between the mobility and origin linear components is in general negative, and thus one can write $(\Gamma_2 - \Gamma_1)(-\omega_{(k,i)}) = (\Gamma_1 - \Gamma_2)\omega_{(k,i)}$ in Equation E.5.

E.5 denote the corresponding parameters from the marginal origin model presented in Equation E.4.

Several points are worth noting regarding Equation E.5. First, the marginal origin slope, or α_M, underlying Equation E.4 is a weighted sum of the ST and the intra-destination slope, with weights given by the relationships between destination and mobility, respectively, with origin. As noted in the main text, the intra-destination slope compares heterogeneous class-mobility groups, conflating structural with dynamic inequalities. For example, within a "middle" destination class, comparing a "low" class group with a "high" class group is simultaneously comparing a group that is upwardly mobile with another that is downwardly mobile. Because the marginal origin slope is in part a function of the intra-destination slope, we generally do not recommend using estimates from the marginal origin model.[15] Second, when the SM slope is zero (i.e., $\Gamma_2 = 0$), then the marginal origin slope will only be a function of the ST slope. In other words, in an absence of any observed social mobility, the marginal origin model will reflect overall structural inequalities. It is in this restricted sense that the marginal origin model could be used.[16] Third, the intercept, marginal origin slope, and origin nonlinearities will all have some degree of bias due to the exclusion of destination and mobility nonlinearities from Equation E.4.[17] Finally, the error term of the marginal origin model will reflect not just individual-level error, but also unique cell-specific heterogeneity as well as additional heterogeneity attributable to class destination and mobility.

ii. Marginal Destination Model

More commonly, researchers frequently use a model that, while including other covariates, only includes class destination, omitting class origin and mobility (e.g., Goldthorpe 1999). The marginal class destination model, arguably the dominant model in sociology and demography, has the following general form:

$$\mu_{rijk} = f(D) = \mu^* + \beta_j^* + \epsilon_{rj}^*, \tag{E.6}$$

where μ^* is the intercept; β_j^* are parameters for class destination using sum-to-zero deviation (or "effect") coding; and ϵ_{rj}^* denotes individual-level error. Using the SDI model as a reference (see Equation E.1), the class destination model outlined in Equation E.6 can be shown to be equivalent to the following:

$$\mu_{rijk} = \underbrace{(\mu + \xi_\mu)}_{\mu^*} + \underbrace{\big(\beta_M + \xi_{\beta_M}\big)(j - j^*) + (\widetilde{\beta}_j + \xi_{\widetilde{\beta}_j})}_{\beta_j^*} + \underbrace{(\epsilon_{rijk} + \eta_{ijk} + \nu_{ijk})}_{\epsilon_{rj}^*} \quad \text{and}$$
$$\beta_M = \big(\Gamma_1 \omega_{(i,j)} + \Gamma_2 \omega_{(k,j)}\big), \tag{E.7}$$

where μ is the intercept; $\Gamma_1 = \alpha + \beta$ is the ST slope and $\Gamma_2 = \gamma + \beta$ is the SM slope; β_M is the marginal destination slope; $\omega_{(i,j)}$ is the relationship between origin linear component and the destination linear component conditional on the intercept and the destination nonlinear components; $\omega_{(k,j)}$ is the relationship between the mobility linear component and the destination linear component conditional on the intercept and the destination nonlinear components; $\widetilde{\beta}_j$ is the jth destination nonlinearity; ξ_μ, ξ_{β_M} and $\xi_{\widetilde{\beta}_j}$ are bias terms for the intercept, marginal destination slope, and

[15] One is generally better off using a model of the form $f(O, M)$. For additional discussion on the merits of models of this general form, see the main text.

[16] Note that, with data organized by origin, destination, and mobility, one can test whether or not the SM slope is zero or not using, for example, the SDI model.

[17] However, because our goal is primarily to conduct a descriptive rather than causal analysis, bias is less of a concern than understanding what, exactly, is being described with a particular model.

jth destination nonlinearity; ϵ_{rijk} is individual-level error; η_{ijk} denotes unique cell-specific heterogeneity; ν_{ijk} denotes additional heterogeneity attributable to class origin and mobility. The terms in brackets below Equation E.7 denote the corresponding parameters from the marginal destination model presented in Equation E.6.

Several main points are particularly noteworthy regarding Equation E.7. First, the marginal destination slope is a weighted sum of the ST and SM slopes, where the weights are given by the relationships between class origin and mobility, respectively, with class destination. Intuitively, this reflects the fact that class destination is a function of both structural and mobility processes. Accordingly, as an overall index of social stratification, class destination is in general a useful and informative metric. Second, the intercept, marginal destination slope, and destination nonlinearities will all have some degree of bias due to the exclusion of origin and mobility nonlinearities from equation E.6. Finally, the error term of the marginal destination model will reflect not only individual-level error, but also unique cell-specific heterogeneity on the mobility table, as well as additional heterogeneity attributable to class origin and mobility.

iii. Marginal Mobility Model

The third possible one-factor class model is the marginal class mobility model (e.g., see Chen et al. 2022), which has the following form:

$$\mu_{rijk} = f(M) = \mu^* + \gamma_k^* + \epsilon_{rk}^*, \tag{E.8}$$

where μ^* is the intercept; γ_k^* are parameters for class mobility using sum-to-zero deviation (or "effect") coding; and ϵ_{rk}^* denotes individual-level error. Using the Diff-SI model as a reference (see Equation E.2), the mobility model outlined in Equation E.8 can be shown to be equivalent to the following:

$$\begin{aligned} \mu_{rijk} &= \underbrace{(\mu + \psi_\mu)}_{\mu^*} + \underbrace{\big(\gamma_M + \psi_{\gamma_M}\big)(k - k^*) + (\widetilde{\gamma}_k + \psi_{\widetilde{\gamma}_k})}_{\gamma_k^*} + \underbrace{(\epsilon_{rijk} + \eta_{ijk} + \nu_{ijk})}_{\epsilon_{rk}^*} \quad \text{and} \\ \gamma_M &= \big((\Gamma_1 - \Gamma_2)\omega_{(i,k)} + \Gamma_2\omega_{(j,k)}\big), \end{aligned} \tag{E.9}$$

where μ is the intercept; $\Gamma_1 - \Gamma_2 = \alpha - \gamma$ is the intra-destination slope and $\Gamma_2 = \gamma + \beta$ is the SM slope; γ_M is the marginal mobility slope; $\omega_{(i,k)}$ is the relationship between origin linear component and the mobility linear component conditional on the intercept and mobility nonlinear components;[18] $\omega_{(j,k)}$ is the relationship between the destination linear component and the mobility linear component conditional on the intercept and mobility nonlinear components; $\widetilde{\gamma}_k$ is the kth mobility nonlinearity; ψ_μ, ψ_{γ_M} and $\psi_{\widetilde{\gamma}_k}$ are bias terms for the intercept, marginal mobility slope, and kth mobility nonlinearity; ϵ_{rijk} is individual-level error; η_{ijk} denotes unique cell-specific heterogeneity; ν_{ijk} denotes additional heterogeneity attributable to class origin and destination. The terms in brackets below Equation E.9 denote the corresponding parameters from the marginal mobility model presented in Equation E.8.

As with the class destination model, several points are worth noting regarding Equation E.9. First, the marginal mobility slope, or γ_M, underlying Equation E.8 is a weighted sum of the intra-destination slope and the SM slope, with weights given by the relationships between origin and destination, respectively, with mobility. Again, as noted when discussing the marginal origin model, the intra-destination slope compares heterogeneous class-mobility groups, conflating structural with dynamic inequalities. Similarly, because the marginal mobility slope is a function of the intra-destination slope, we generally do not recommend using estimates from the marginal mobility

[18] Similar to the corresponding weight in Equation E.5, the relationship between the origin and mobility linear components is generally negative, and thus one can write $(\Gamma_1 - \Gamma_2)(-\omega_{(i,k)}) = (\Gamma_2 - \Gamma_1)\omega_{(i,k)}$ in Equation E.9.

model. Second, when the ST slope is zero (i.e., $\Gamma_1 = 0$), then the marginal mobility slope will only be a function of the SM slope. In other words, in an absence of any overall total social structural differences, the marginal mobility model will reflect the observed mobility patterns. It is in this restricted sense that the marginal mobility model could be used.[19] Third, as with the marginal destination model, the intercept, marginal mobility slope, and mobility nonlinearities will all have some degree of bias due to the exclusion of origin and destination nonlinearities from Equation E.8. Lastly, the error term of the marginal mobility model will capture not just individual-level error, but also unique cell-specific heterogeneity as well as additional heterogeneity attributable to class origin and destination.

3. Interpreting the Parameters of Two-Factor Class Models

In this section, we clarify the interpretation of the parameters from all three logically possible two-factor models (origin-destination, destination-mobility, origin-mobility). As with the one-factor models, we present each two-factor model and discuss the relationship between each model's parameters and those of a corresponding model that includes all three factors (see Equations E.1, E.2, and E.3). Again, to avoid confusion with corresponding terms in the three-factor models outlined earlier, we use asterisks to denote parameters from two-factor models. It should be emphasized that we treat these models as descriptive, not causal. In general, our analyses suggest that among the two-factor models, an origin-mobility model is preferable to an origin-destination or destination-mobility model. This is because the underlying linear terms of the origin-mobility model are the ST and SM slopes, which estimate structural and dynamic inequalities, whereas the other two models generate estimates of within-destination differences that compare heterogeneous origin-mobility groups.

i. Origin-Destination Model

The origin-destination model, also known as the "square additive model" (Hope 1971, 1975), has the following general form (cf. Duncan 1966: 94-95):

$$\mu_{rijk} = f(O, D) = \mu^* + \alpha_i^* + \beta_j^* + \eta_{ij}^* + \epsilon_{rij}^*, \tag{E.10}$$

where μ^* is the intercept; α_i^* and β_j^* are parameters for origin and destination using sum-to-zero deviation (or "effect") coding; η_{ij}^* denotes group-level heterogeneity terms;[20] and ϵ_{rij}^* denotes individual-level error. Using the Diff-SI model (see Equation E.2), the origin-destination model outlined in Equation E.10 can be shown to be equivalent to the following:

[19] Note that, with data organized by origin, destination, and mobility, one can test whether or not the ST slope is zero or not using, for example, the SDI model.

[20] Following Duncan (1966: 94-95), we will treat the group-level heterogeneity terms for all of the two-factor models as orthogonal. These terms can be easily calculated as group-level residuals relative to a fully-saturated model. With respect to the origin-destination model, as an alternative one can specify all possible pairs of interactions between origin and destination. If the data are balanced such that there are an equal number of individual-level observations in each origin-destination cell, then, using sum-to-zero deviation coding or orthogonal polynomial coding, the residuals will be equivalent to specifying a full set of origin-destination interactions. The reason for this is that in such a setting the columns for the origin-destination interactions will be orthogonal to the main origin and destination columns.

$$\mu_{rijk} = \underbrace{(\mu + \psi_\mu)}_{\mu^*} + \underbrace{\big((\Gamma_1 - \Gamma_2) + \psi_{(\Gamma_1-\Gamma_2)}\big)(i - i^*) + (\widetilde{\alpha}_i + \psi_{\widetilde{\alpha}_i})}_{\alpha_i^*}$$

$$+ \underbrace{(\Gamma_2 + \psi_{\Gamma_2})(j - j^*) + (\widetilde{\beta}_j + \psi_{\widetilde{\beta}_j})}_{\beta_j^*} + \underbrace{(\eta_{ijk} + \nu_{ijk})}_{\eta_{ij}^*} + \underbrace{\epsilon_{rijk}}_{\epsilon_{rij}^*}, \qquad \text{(E.11)}$$

where μ is the intercept; $\Gamma_1 - \Gamma_2 = \alpha - \gamma$ is a slope of intra-destination differences; Γ_2 is the SM slope; $\widetilde{\alpha}_i$ is the ith origin nonlinearity; $\widetilde{\beta}_j$ is the jth destination nonlinearity; ψ_μ, $\psi_{(\Gamma_1-\Gamma_2)}$, ψ_{Γ_2}, $\psi_{\widetilde{\alpha}_i}$, and $\psi_{\widetilde{\beta}_j}$ are bias terms for the intercept, intra-destination slope, SM slope, ith origin nonlinearity, and jth destination nonlinearity; η_{ijk} denotes terms for unique cell-specific heterogeneity; ν_{ijk} denotes terms for unique mobility-attributed heterogeneity; and ϵ_{rijk} is individual-level error. The terms in brackets below Equation E.11 denote the corresponding parameters from the origin-destination model presented in Equation E.10.

Three main points stand out from Equation E.11. First, the intercept, origin, and destination parameters will all have some degree of bias due to the exclusion of the mobility nonlinear components from the origin-destination model.[21] Second, assuming that there is no bias due to the exclusion of the mobility nonlinearities, either because the mobility nonlinearities are zero or the mobility variables are unrelated to the variables for the intercept, origin, and destination terms (i.e., the included variables), then the underlying origin and destination slopes of the origin-destination model will equal those from the Diff-SI model. In other words, the origin-destination model will generate a slope of intra-destination differences. For this reason we do not generally recommend using the origin-destination model without extreme care in the interpretation of the origin parameters in Equation E.10. Finally, the group-level heterogeneity terms η_{ij}^* from the origin-destination model equal the sum of the unique cell-specific heterogeneity terms from the Diff-SI model, η_{ijk}, and the unique mobility-attributed heterogeneity terms ν_{ijk}. The mobility-attributed heterogeneity terms are simply the predicted values from the parameters for the mobility nonlinear components (i.e., the excluded variables) using that part of the mobility variables that is unassociated with the variables for the intercept, origin, and destination terms (i.e., the included variables).

ii. Destination-Mobility Model

The second logically possible two-factor model is the destination-mobility model, which has the following general form:

$$\mu_{rijk} = \mu^* + \beta_j^* + \gamma_k^* + \eta_{jk}^* + \epsilon_{rjk}^*, \qquad \text{(E.12)}$$

where μ^* is the intercept; β_j^* and γ_k^* are parameters for destination and mobility using sum-to-zero deviation coding; η_{jk}^* denotes group-level heterogeneity terms; and ϵ_{rjk}^* is individual-level error. Using the DI-Diff model (see Equation E.3), the destination-mobility model outlined in Equation E.12 can be shown to be equal to the following:

[21] Note that the individual-level error term is unbiased. The reason for this is that the origin-destination model with the group-level heterogeneity terms is saturated, so the individual-level error will be the same as that from the Diff-SI model with unique heterogeneity terms, which is also saturated. Again, because our focus here is on descriptive rather than causal models, we are less concerned about the parameters being biased than that researchers have a clear idea of what is being estimated in the models.

$$\mu_{rijk} = \underbrace{(\mu + \phi_\mu)}_{\mu^*} + \underbrace{(\Gamma_1 + \phi_{\Gamma_1})(j - j^*) + (\widetilde{\beta}_j + \phi_{\widetilde{\beta}_j})}_{\beta_j^*}$$
$$+ \underbrace{\big((\Gamma_2 - \Gamma_1) + \phi_{(\Gamma_2 - \Gamma_1)}\big)(k - k^*) + (\widetilde{\gamma}_k + \phi_{\widetilde{\gamma}_k})}_{\gamma_k^*} + \underbrace{(\eta_{ijk} + \nu_{ijk})}_{\eta_{jk}^*} + \underbrace{\epsilon_{rijk}}_{\epsilon_{rjk}^*}, \qquad \text{(E.13)}$$

where μ is the intercept; $\Gamma_1 = \alpha + \beta$ is the ST slope; $\Gamma_2 - \Gamma_1 = \gamma - \alpha$ is the intra-destination slope; $\widetilde{\beta}_j$ is the jth destination nonlinearity; $\widetilde{\gamma}_k$ is the kth mobility nonlinearity; ϕ_μ, ϕ_{Γ_1}, $\phi_{(\Gamma_2-\Gamma_1)}$, $\phi_{\widetilde{\beta}_j}$, and $\phi_{\widetilde{\gamma}_k}$ are bias terms for the intercept, ST slope, intra-destination slope, jth destination nonlinearity, and kth mobility nonlinearity; η_{ijk} denotes terms for unique cell-specific heterogeneity; ν_{ijk} denotes terms for unique origin-attributed heterogeneity; and ϵ_{rijk} is individual-level error. The terms in brackets below Equation E.13 denote the corresponding parameters from the destination-mobility model displayed in Equation E.12.

As with the origin-destination model, there are three main conclusions that follow from Equation E.13. First, as indicated by the presence of the ϕ parameters, the intercept, destination, and mobility parameters will be biased because of the exclusion of the origin nonlinearities from the destination-mobility model. Second, assuming that excluding the mobility nonlinearities results in no bias, then the underlying destination and mobility slopes will equal those from the DI-Diff model. Lastly, the group-level heterogeneity terms η_{jk}^* equal the sum of the unique cell-specific heterogeneity terms from the DI-Diff model, η_{ijk}, and the unique origin-attributed heterogeneity terms ν_{ijk}. Similar to the origin-destination model, the origin-attributed heterogeneity terms are just the predicted values from the parameters for the origin nonlinear components (i.e., the excluded variables) using that part of the origin variables that is unrelated to the variables for the intercept, destination, and mobility terms (i.e., the included variables).

iii. Origin-Mobility Model

The remaining two-factor model is the origin-mobility model, which has the following general form:

$$\mu_{rijk} = \mu^* + \alpha_i^* + \gamma_k^* + \eta_{ik}^* + \epsilon_{rik}^*, \qquad \text{(E.14)}$$

where μ^* is the intercept; α_i^* and γ_k^* are parameters for origin and mobility using sum-to-zero deviation coding; η_{ik}^* denotes group-level heterogeneity terms; and ϵ_{rik}^* is individual-level error. Using the SDI model (see Equation E.1), the origin-mobility model presented in Equation E.14 is shown to be equivalent to the following:

$$\mu_{rijk} = \underbrace{(\mu + \xi_\mu)}_{\mu^*} + \underbrace{(\Gamma_1 + \xi_{\Gamma_1})(i - i^*) + (\widetilde{\alpha}_i + \xi_{\widetilde{\alpha}_i})}_{\alpha_i^*}$$
$$+ \underbrace{\big(\Gamma_2 + \xi_{\Gamma_2}\big)(k - k^*) + (\widetilde{\gamma}_k + \xi_{\widetilde{\gamma}_k})}_{\gamma_k^*} + \underbrace{(\eta_{ijk} + \nu_{ijk})}_{\eta_{ik}^*} + \underbrace{\epsilon_{rijk}}_{\epsilon_{rik}^*}, \qquad \text{(E.15)}$$

where μ is the intercept; $\Gamma_1 = \alpha + \beta$ is the ST slope; $\Gamma_2 = \gamma + \beta$ is the SM slope; $\widetilde{\alpha}_i$ is the ith origin nonlinearity; $\widetilde{\gamma}_k$ is the kth mobility nonlinearity; ξ_μ, ξ_{Γ_1}, ξ_{Γ_2}, $\xi_{\widetilde{\alpha}_i}$, and $\xi_{\widetilde{\gamma}_k}$ are bias terms for the intercept, ST slope, SM slope, ith origin nonlinearity, and kth mobility nonlinearity; η_{ijk} denotes terms for unique cell-specific heterogeneity; ν_{ijk} denotes terms for unique destination-attributed heterogeneity; and ϵ_{rijk} is individual-level error. The terms in brackets below Equation E.15 refer to the corresponding parameters from the origin-mobility model shown in Equation E.14.

As with the other two-factor models, there are three main takeaways from Equation E.15. First, the intercept, origin, and mobility parameters will be biased because of the exclusion of the destination nonlinearities from the origin-mobility model. Second, assuming that excluding the destination nonlinearities produces no bias, then the underlying origin and mobility slopes will equal those from the SDI model. Finally, the group-level heterogeneity terms η^*_{ik} equal the sum of the unique cell-specific heterogeneity terms from the SDI model, η_{ijk}, and the unique destination-attributed heterogeneity terms ν_{ijk}. The destination-attributed heterogeneity terms are, like those for the other two-factor models, simply the predicted values from the parameters for the destination nonlinearities (i.e., the excluded variables) using that part of the destination variables that is unrelated to the variables for the intercept, origin, and mobility terms (i.e., the included variables).

4. Derivation of Relationships

In this section we show how the relationships outlined above can be derived using matrix algebra and the logic of omitted variable bias. We first present the derivation for the one-factor formulas using the one-factor destination model as an example. Next, we show the derivation for the two-factor models using the origin-destination model (i.e., Duncan's "square additive model").

i. Derivation for One-Factor Models

To show the derivation for the one-factor models, we use the class destination model, but similar calculations can be applied to derive the one-factor origin and mobility models. Suppose we fit the one-factor destination model (Equation E.6) on an individual-level data set indexed by origin, destination, and mobility. To reveal the underlying structure of the model, it is useful to express Equation E.6 as a linearized destination model, which decomposes each deviation from the overall mean into its constituent linear and nonlinear components:

$$\mu_{rijk} = f(D) = \mu^* + \beta^*(j - j^*) + \widetilde{\beta}^*_j + \epsilon^*_{rj}, \tag{E.16}$$

where the parameters are the same as in Equation E.6 except β^* denotes the destination slope and $\widetilde{\beta}^*_j$ the jth destination deviation from the overall mean. Because only the coding scheme differs between Equation E.16 and Equation E.6, we will refer to them interchangeably as a class destination model in the following discussion.

Let $\mathbf{y}$ denote an $R \times 1$ column vector of outcome values (e.g., means), $\mathbf{1}$ an $R \times 1$ column vector of 1's, $\mathbf{d}_L$ an $R \times 1$ column vector of the class destination linear component, and $\widetilde{\mathbf{D}}$ an $R \times (J-2)$ matrix of orthogonal destination polynomials with no linear component. Using matrix notation, the marginal destination model in Equation E.16 can be expressed as follows:

$$\mathbf{y} = \mathbf{1}\mu^* + \mathbf{d}_L\beta^*_L + \widetilde{\mathbf{D}}\widetilde{\boldsymbol{\beta}}^* + \boldsymbol{\epsilon}^*. \tag{E.17}$$

where μ^* is again the intercept, β^*_L is the estimated marginal destination slope, $\widetilde{\boldsymbol{\beta}}^*$ is a $(J-2) \times 1$ column vector of nonlinear destination parameters, and $\boldsymbol{\epsilon}^*$ is an $R \times 1$ column vector of individual-level error terms.

For the purposes of comparison, note that the SDI model (see Equation E.1) can be specified in matrix form as:

$$\mathbf{y} = \mathbf{1}\mu + \mathbf{O}\boldsymbol{\alpha} + \mathbf{M}\boldsymbol{\gamma} + \widetilde{\mathbf{D}}\tilde{\boldsymbol{\beta}} + \boldsymbol{\eta} + \boldsymbol{\epsilon}, \tag{E.18}$$

where μ is the intercept, $\mathbf{O}$ is an $R \times (I-1)$ matrix of orthogonal origin polynomials, $\boldsymbol{\alpha}$ is an $(I-1) \times 1$ column vector of linear and nonlinear origin parameters, $\mathbf{M}$ is an $R \times (K-1)$ matrix of orthogonal mobility polynomials, $\boldsymbol{\gamma}$ is a $(K-1) \times 1$ column vector of linear and nonlinear mobility parameters, $\widetilde{\mathbf{D}}$ is an $R \times (J-2)$ matrix of orthogonal destination polynomials without the linear component, $\widetilde{\boldsymbol{\beta}}$ is a $(J-2)\times 1$ column vector of nonlinear destination parameters, $\boldsymbol{\eta}$ is an $R\times 1$ column vector of cell-specific heterogeneity terms, and $\boldsymbol{\epsilon}$ is an $R\times 1$ column vector of individual-level error terms.

To clarify the interpretation of the marginal destination model, we need to specify an auxiliary equation that expresses the association between those variables included in the marginal destination model and those excluded from the marginal destination model but included in the SDI model. To do so, we first define a matrix $\mathbf{S}$ of dimension $J\times(I+K-2)$ as follows:

$$\mathbf{S}=(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z}, \tag{E.19}$$

where $\mathbf{X}=[\mathbf{1}\,\mathbf{d}_L\,\widetilde{\mathbf{D}}]$ is an $R\times J$ matrix of 1's, the destination linear component, and higher-order orthogonal destination polynomials; and $\mathbf{Z}=[\mathbf{O},\mathbf{M}]$ is an $R\times(I+K-2)$ matrix of orthogonal origin and mobility polynomials. The matrix $\mathbf{S}$ is simply a collection of parameters representing relationships between those variables included in the marginal destination model ($\mathbf{X}$) and those variables excluded from the marginal destination model but included in the SDI model ($\mathbf{Z}$). Using $\mathbf{Z}$, $\mathbf{S}$, and $\mathbf{X}$, we can accordingly define an auxiliary equation compactly as $\mathbf{Z}=\mathbf{XS}+\mathbf{U}_Z$ or, equivalently:

$$[\mathbf{O},\mathbf{M}]=\mathbf{1}\mathbf{s}_\mu+\mathbf{d}_L\mathbf{s}_{d_L}+\widetilde{\mathbf{D}}\mathbf{S}_{\widetilde{D}}+\mathbf{U}_{[O,M]}, \tag{E.20}$$

where $\mathbf{s}_\mu$ is a $1\times(I+K-2)$ row vector of parameters, $\mathbf{s}_{d_L}$ is a $1\times(I+K-2)$ row vector of parameters, $\mathbf{S}_{\widetilde{D}}$ is a $(J-2)\times(I+K-2)$ matrix of parameters, and $\mathbf{U}_{[O,M]}$ is an $R\times(I+K-2)$ matrix of error terms representing that part of $\mathbf{O}$ and $\mathbf{M}$ unrelated to the variables included in the marginal destination model (i.e., the intercept, destination linear component, and higher-order destination polynomials).[22]

To clarify the meaning of the parameters of the marginal destination model (Equation E.17), we can simply substitute Equation E.20 into Equation E.17. This is easily accomplished by re-writing Equation E.18 as $\mathbf{y}=\mathbf{1}\mu+\mathbf{Z}\boldsymbol{\zeta}+\widetilde{\mathbf{D}}\widetilde{\boldsymbol{\beta}}+\boldsymbol{\eta}+\boldsymbol{\epsilon}$, where $\boldsymbol{\zeta}$ is an $(I+K-2)\times 1$ column vector of origin and mobility parameters such that:

$$\boldsymbol{\zeta}=\begin{pmatrix}\boldsymbol{\alpha}\\ \boldsymbol{\gamma}\end{pmatrix}$$

We then just plug in $\mathbf{Z}=\mathbf{1}\mathbf{s}_\mu+\mathbf{d}_L\mathbf{s}_{d_L}+\widetilde{\mathbf{D}}\mathbf{S}_{\widetilde{D}}+\mathbf{U}_Z$ into this equation. After rearranging terms, we obtain the following:

$$\mathbf{y}=\mathbf{1}\underbrace{(\mu+\mathbf{s}_\mu\boldsymbol{\zeta})}_{\mu^*}+\mathbf{d}_L\underbrace{(\mathbf{s}_{d_L}\boldsymbol{\zeta})}_{\beta_L^*}+\widetilde{\mathbf{D}}\underbrace{(\widetilde{\boldsymbol{\beta}}+\mathbf{S}_{\widetilde{D}}\boldsymbol{\zeta})}_{\widetilde{\boldsymbol{\beta}}^*}+\underbrace{\boldsymbol{\epsilon}+\boldsymbol{\eta}+\mathbf{U}_Z\boldsymbol{\zeta}}_{\boldsymbol{\epsilon}^*}, \tag{E.21}$$

which reveals how the SDI model is related to the marginal destination model. Several points are worth emphasizing. First, the marginal destination slope is a weighted sum of the ST and SM slopes (which are contained in $\boldsymbol{\zeta}$), with weights given by the relationships between the destination linear component and the origin and mobility linear components (which are contained in the row vector $\mathbf{s}_{d_L}$). Second, the parameters from the marginal destination model will all have some degree of bias due to the exclusion of the origin and mobility components. Depending on the structure of the data, the origin and mobility polynomials in $\mathbf{Z}$ will be more or less associated with the set of included variables, namely, the vector $\mathbf{1}$, destination linear component $\mathbf{d}_L$, and higher-order destination polynomials $\widetilde{\mathbf{D}}$.[23] If these relationships are strong, then the bias will be large, and the parameter

[22] Note that $\mathbf{s}_\mu$ is simply the first row of $\mathbf{S}$, $\mathbf{s}_{d_L}$ is the second row, and $\mathbf{S}_{\widetilde{D}}$ is rows 3 to J of $\mathbf{S}$.

[23] However, note that, because the marginal destination slope is defined by the weighted sum of the ST and SM slopes, the bias for the marginal destination slope is a function of only the relationship between the included variables and the

estimates for the intercept and destination nonlinear terms from the marginal destination model and the SDI models will differ, possibly quite substantially. By contrast, if these relationships are weak, then the bias will be relatively small, such that the intercept and destination nonlinear terms of the marginal destination model will be approximately equal to those from the SDI model. Lastly, the individual-level error term ($\boldsymbol{\epsilon}^*$) of the marginal destination model can be interpreted as the sum of individual-level error terms from the SDI model, the unique cell-specific heterogeneity terms $\boldsymbol{\eta}$, and the column vector of origin and mobility parameters $\boldsymbol{\zeta}$, the latter of which are weighted by $\mathbf{U}_Z$, or that part of the excluded variables (i.e., the orthogonal origin and mobility polynomials) unrelated to the variables included in the marginal destination model.

ii. Derivation for Two-Factor Models

We illustrate the derivation for the two-factor models using the origin-destination model, but similar calculations can be applied to the destination-mobility and origin-mobility models. Suppose we fit the origin-destination model (Equation E.10) on an individual-level data set indexed by origin, destination, and mobility. To reveal the underlying structure of the model, it is useful to express Equation E.10 as a linearized origin-destination model with group-level heterogeneity terms, which decomposes each deviation from the overall mean into its constitutive linear and nonlinear components:

$$\mu_{rijk} = \mu^* + \alpha^*(i - i^*) + \widetilde{\alpha}_i^* + \beta^*(j - j^*) + \widetilde{\beta}_j^* + \eta_{ij}^* + \epsilon_{rij}^*, \tag{E.22}$$

where the parameters are the same as in Equation E.10 except α^* denotes the origin slope, $\widetilde{\alpha}_i^*$ the ith origin deviation from the overall mean, β^* the destination slope, and $\widetilde{\beta}_j^*$ the jth destination deviation from the overall mean. Because Equation E.22 is the same as that in Equation E.10, but with a different coding scheme, we will refer to them interchangeably as an origin-destination model in the discussion that follows.

Let $\mathbf{y}$ denote an $R \times 1$ column vector of outcome values (e.g., means), $\mathbf{1}$ an $R \times 1$ column vector of 1's, $\mathbf{O}$ an $R \times (I-1)$ matrix of orthogonal origin polynomials, and $\mathbf{D}$ an $R \times (J-1)$ matrix of orthogonal destination polynomials. Using matrix notation, the origin-destination model in Equation E.22 can be expressed as follows:

$$\mathbf{y} = \mathbf{1}\mu^* + \mathbf{O}\boldsymbol{\alpha}^* + \mathbf{D}\boldsymbol{\beta}^* + \boldsymbol{\eta}^* + \boldsymbol{\epsilon}^*. \tag{E.23}$$

where μ^* is again the intercept, $\boldsymbol{\alpha}^*$ is an $(I-1) \times 1$ column vector of linear and nonlinear origin parameters, $\boldsymbol{\beta}^*$ is a $(J-1) \times 1$ column vector of linear and nonlinear destination parameters, $\boldsymbol{\eta}^*$ is an $R \times 1$ column vector of group-level heterogeneity parameters, and $\boldsymbol{\epsilon}^*$ is an $R \times 1$ column vector of individual-level error terms.

For the purposes of comparison, note that the Diff-SI model (see Equation E.2) can be specified in matrix form as:

$$\mathbf{y} = \mathbf{1}\mu + \mathbf{O}\boldsymbol{\alpha} + \mathbf{D}\boldsymbol{\beta} + \widetilde{\mathbf{M}}\widetilde{\boldsymbol{\gamma}} + \boldsymbol{\eta} + \boldsymbol{\epsilon}, \tag{E.24}$$

where μ is the intercept, $\boldsymbol{\alpha}$ is an $(I-1)\times 1$ column vector of linear and nonlinear origin parameters, $\boldsymbol{\beta}$ is a $(J-1)\times 1$ column vector of linear and nonlinear destination parameters, $\widetilde{\mathbf{M}}$ an $R \times (K-2)$ matrix of orthogonal mobility polynomials with no linear component, $\widetilde{\boldsymbol{\gamma}}$ is a $(K-2) \times 1$ column vector of nonlinear mobility parameters, $\boldsymbol{\eta}$ is an $R \times 1$ column vector of cell-specific heterogeneity terms, and $\boldsymbol{\epsilon}$ is an $R \times 1$ column vector of individual-level error terms.

Similar to the calculations for the marginal destination model in the previous section, to interpret the meaning of the parameters of the origin-destination model, we need to specify an auxiliary equation that expresses the association between those variables included in the origin-destination

origin and mobility polynomials without the linear component.

model and those excluded from the origin-destination model but included in the Diff-SI model. As before, we can define a matrix $\mathbf{S}$ of dimension $(I+J-1)\times(K-2)$ as $\mathbf{S}=(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z}$, where $\mathbf{X}=[\mathbf{1}\ \mathbf{O}\ \mathbf{D}]$ is an $R\times(I+J-1)$ matrix of 1's, orthogonal origin polynomials, and orthogonal destination polynomials; and $\mathbf{Z}=\widetilde{\mathbf{M}}$ is an $R\times(K-2)$ matrix of orthogonal class mobility polynomials with no linear component. Again, the matrix $\mathbf{S}$ is simply a collection of parameters representing relationships between those variables included in the origin-destination model ($\mathbf{X}$) and those variables excluded from the origin-destination model but included in the Diff-SI model ($\mathbf{Z}$). Using $\mathbf{Z}$, $\mathbf{S}$, and $\mathbf{X}$, we can accordingly define an auxiliary equation compactly as $\mathbf{Z}=\mathbf{XS}+\mathbf{U}_Z$ or, equivalently:

$$\widetilde{\mathbf{M}}=\mathbf{1s}_\mu+\mathbf{OS}_O+\mathbf{DS}_D+\mathbf{U}_{\widetilde{M}}, \tag{E.25}$$

where $\mathbf{s}_\mu$ is a $1\times(K-2)$ row vector of parameters, $\mathbf{S}_O$ is an $(I-1)\times(K-2)$ matrix of parameters, $\mathbf{S}_D$ is a $(J-1)\times(K-2)$ matrix of parameters, and $\mathbf{U}_{\widetilde{M}}$ is an $R\times(K-2)$ matrix of error terms representing that part of $\widetilde{\mathbf{M}}$ unrelated to the variables included in the origin-destination model.[24]

To clarify the meaning of the parameters of the origin-destination model (Equation E.22), we can simply substitute Equation E.25 into Equation E.23. After substituting and rearranging terms, we obtain the following equation:

$$\mathbf{y}=\mathbf{1}\underbrace{(\mu+\mathbf{s}_\mu\tilde{\boldsymbol{\gamma}})}_{\mu^*}+\mathbf{O}\underbrace{(\boldsymbol{\alpha}+\mathbf{S}_O\tilde{\boldsymbol{\gamma}})}_{\boldsymbol{\alpha}^*}+\mathbf{D}\underbrace{(\boldsymbol{\beta}+\mathbf{S}_D\tilde{\boldsymbol{\gamma}})}_{\boldsymbol{\beta}^*}+\underbrace{(\boldsymbol{\eta}+\mathbf{U}_{\widetilde{M}}\tilde{\boldsymbol{\gamma}})}_{\boldsymbol{\eta}^*}+\underbrace{\boldsymbol{\epsilon}}_{\boldsymbol{\epsilon}^*}, \tag{E.26}$$

which reveals how the Diff-SI model is related to the origin-destination model. As noted previously, the intercept, origin, and destination parameters from the origin-destination model will all have some degree of bias due to the exclusion of the mobility nonlinear components. Depending on the structure of the data, the orthogonal mobility polynomials in $\widetilde{\mathbf{M}}$ will be more or less related to the vector $\mathbf{1}$, orthogonal origin polynomials $\mathbf{O}$, and orthogonal destination polynomials $\mathbf{D}$. If these relationships are strong, then the bias will be large, and the parameter estimates from the origin-destination and the Diff-SI models will differ, possibly quite substantially. By contrast, if these relationships are weak, then the bias will be relatively small, such that the intercept, origin, and destination parameters of the origin-destination model will be approximately equal to those from the Diff-SI model. Similarly, the vector of group-level heterogeneity terms $\boldsymbol{\eta}^*$ in the origin-destination model, which can be interpreted as a restricted set of origin-destination interactions, is equal to a weighted sum of the cell-specific heterogeneity terms $\boldsymbol{\eta}$ and the mobility nonlinearities $\widetilde{\boldsymbol{\gamma}}$, the latter of which are weighted by $\mathbf{U}_{\widetilde{M}}$, or that part of the excluded variables unrelated to the variables included in the origin-destination model.

Equation E.26 additionally clarifies how the mobility nonlinearities can be viewed as a kind of "structured" interaction with respect to origin and destination.[25] We can show this relationship by taking the equation $\boldsymbol{\eta}^*=\boldsymbol{\eta}+\mathbf{U}_{\widetilde{M}}\widetilde{\boldsymbol{\gamma}}$ and solving for $\widetilde{\boldsymbol{\gamma}}$, the mobility nonlinearities from the Diff-SI model. Because $\mathbf{U}_{\widetilde{M}}$ is non-square, it does not have a regular inverse. However, it has a Moore-Penrose generalized inverse that is equal to the left inverse of $\mathbf{U}_{\widetilde{M}}$. Solving for $\widetilde{\boldsymbol{\gamma}}$ gives us the following:

$$\begin{aligned}\widetilde{\boldsymbol{\gamma}}&=\mathbf{U}^{+}_{\widetilde{M}}(\boldsymbol{\eta}^*-\boldsymbol{\eta})=(\mathbf{U}'_{\widetilde{M}}\mathbf{U}_{\widetilde{M}})^{-1}\mathbf{U}'_{\widetilde{M}}(\boldsymbol{\eta}^*-\boldsymbol{\eta})\\&=(\mathbf{U}'_{\widetilde{M}}\mathbf{U}_{\widetilde{M}})^{-1}\mathbf{U}'_{\widetilde{M}}\boldsymbol{\eta}^*\end{aligned} \tag{E.27}$$

[24]Note that $\mathbf{s}_\mu$ is simply the first row of $\mathbf{S}$, $\mathbf{S}_O$ is rows 2 to I of $\mathbf{S}$, and $\mathbf{S}_D$ is rows $I+1$ to $I+J-1$ of $\mathbf{S}$.

[25]What this means in practice is that the SDI model outlined in the main text is, in fact, interactive in the data although it is additive in the parameters.

where the plus $+$ denotes a Moore-Penrose generalized inverse and $(\mathbf{U}'_{\widetilde{M}}\mathbf{U}_{\widetilde{M}})^{-1}\mathbf{U}'_{\widetilde{M}}$ is the left inverse of $\mathbf{U}_{\widetilde{M}}$.[26] Equation E.27 reveals that the mobility nonlinearities are equal to a regression model predicting heterogeneity from origin-destination interactions using that part of the orthogonal mobility polynomials unrelated to the intercept, origin, and destination variables included in the origin-destination model.[27] Similar derivations as those outlined in this section can be conducted in an analogous way for the destination-mobility and origin-mobility models.[28]

5. Higher-Order Interactions on a Mobility Table

So far little has been stated about the structure of the higher-order interactions beyond that for origin, destination, and mobility. In this section we discuss higher-order interactions on mobility tables. As we illustrate, only a limited number of interactions can be included beyond the main set of parameters for origin, destination, and mobility. This reflects the fact that, descriptively, including all three main parameters means there is already a structured interaction captured by a three-factor model.

To illustrate the limited number of interactions that can be explicitly included, suppose we use orthogonal polynomial contrasts so that we have $I-2$, $J-2$, and $K-2$ columns for the higher-order terms of origin, destination, and mobility, respectively. Then the SDI model[29] can be represented as:

$$Y_{rijk} = \mu + (\alpha+\beta)o_L + (\gamma+\beta)m_L + \sum_{i=2}^{I-1}\alpha^i o_i + \sum_{j=2}^{J-1}\beta^j d_j + \sum_{k=2}^{K-1}\gamma^k m_k + \eta_{ijk} + \xi_{rijk}, \tag{E.28}$$

where η_{ijk} are cell-specific heterogeneity terms and ξ_{rijk} are individual-level errors. We treat these as residual (or orthogonal) to the main parameters in the model in the discussions above (see also Duncan 1966).

However, an alternative representation of the η_{ijk} terms is to specify them as higher-order interactions. However, because of the linear dependency among the variables, only a restricted set of interactions can be included. Specifically, given an origin-destination mobility table, one can specify the η_{ijk} terms as follows:

$$\eta_{ijk} = \sum_{j=2}^{J-1}\beta_{Lj}\,(o_L d_j) \; + \; \sum_{i=2}^{I-2}\sum_{j=2}^{J-1}\beta_{ij}\,(o_i d_j). \tag{E.29}$$

where the number of additional parameters above the baseline SDI model are $(I-2)(J-2)$. Note that these additional terms represent interactions of a "smoothed" origin curve with the destina-

[26] Note that we can drop $\boldsymbol{\eta}$ from Equation E.27 because, by construction (see Equation E.1), it is unrelated to the orthogonal mobility polynomials such that $(\mathbf{U}'_{\widetilde{M}}\mathbf{U}_{\widetilde{M}})^{-1}\mathbf{U}'_{\widetilde{M}}\boldsymbol{\eta}$ will produce a $K-2$ column vector of zeros.

[27] The more the mobility variables are related to the heterogeneity from the origin-destination interactions, the larger in absolute value the size of the mobility nonlinearities. Note further that if the mobility variables are unrelated to the variables included in the origin-destination model, then $\mathbf{U}_{\widetilde{M}} = \widetilde{\mathbf{M}}$. Accordingly, Equation E.27 simplifies further to $(\widetilde{\mathbf{M}}'\widetilde{\mathbf{M}})^{-1}\widetilde{\mathbf{M}}'\boldsymbol{\eta}^*$. In other words, assuming the included and excluded variables are unrelated, we can simply take the mobility variables and use them to predict the heterogeneity from the origin-destination interactions to obtain the mobility nonlinearities. To the extent that the mobility variables are only weakly related to the intercept, origin, and destination variables in the origin-destination model, then this procedure will reproduce, within an error of approximation, the mobility nonlinearities from the Diff-SI model.

[28] However, note that, because there is inherent censoring on a mobility table with respect to mobility, one cannot include all mathematically pairwise interactions in a destination-mobility or origin-mobility model. By contrast, although we have treated origin-destination interactions as cell-specific residuals, one could model them as all possible pairwise interactions on a mobility table.

[29] We use the SDI model for illustrative purposes here, but our results apply to any of the models discussed above.

tion nonlinear components. These interactions could be reversed so that the parameters represent interactions of a "smoothed" destination curve with the origin nonlinear components.

To illustrate how the additional terms can be included in an L-ODM model, consider data from an origin-destination mobility table. Given a mobility table, one can incorporate additional heterogeneity using the particular set of origin-destination interactions outlined in Equation E.29. The β_{Lj} parameters are interaction terms between the origin linear component and the destination nonlinear components, while the β_{ij} parameters are interactions between the origin nonlinear components (except for the last origin nonlinear component) and the destination nonlinear components. This allows a smoothed origin curve to vary as a function of the destination nonlinear components.

To show what the full SDI model with higher-order origin-destination interactions would be, suppose there are $I = 5$ origin groups and $J = 5$ destination groups (and thus $K = I + J - 1 = 9$ mobility groups). Above the baseline SDI model, we can include $(I-2)(J-2) = 9$ additional parameters representing particular origin-destination interactions. Then the SDI model with fully specified higher-order origin-destination interactions is:

$$\begin{aligned}\mu_{rijk} = \mu &+ (\gamma+\beta)m_L + (\alpha+\beta)o_L + \alpha^2 o_2 + \cdots + \alpha^4 o_4 + \beta^2 d_2 + \cdots + \beta^4 d_4 + \gamma^2 m_2 + \cdots + \gamma^8 m_8 + \\ &\beta_{L2}(o_L d_2) + \beta_{L3}(o_L d_3) + \beta_{L4}(o_L d_4) + \beta_{22}(o_2 d_2) + \beta_{23}(o_2 d_3) + \beta_{24}(o_2 d_4) + \\ &\beta_{32}(o_3 d_2) + \beta_{33}(o_3 d_3) + \beta_{34}(o_3 d_4) + \xi_{rijk},\end{aligned} \tag{E.30}$$

where the additional terms represent intra-mobility heterogeneity, or heterogeneity within the diagonals of the mobility table. However, we caution against indiscriminately including these higher-order terms directly in the main model without checking for multicollinearity. In general, including these additional interactions results in a full-rank design matrix (and thus the model is identified), but in practice these additional columns are highly collinear with the main columns of the baseline SDI model, resulting in highly unstable estimates.

Online Appendix F: Additional Structural and Dynamic Inequalities

In the main manuscript, we focus on several key parametric expressions that can be derived from the structural and dynamic inequality (SDI) model. Below, we show a more complete list of expressions that can be derived from the SDI model and then illustrate the analytic use of one additional expression that focuses on class destination.

Full list of expressions derived from the SDI model

Table F.1: Summarizing Structural and Dynamic Inequalities on a Mobility Table

General Terminology	Specific Summary	Mathematical Expression	
Structural Inequality	Social Structure Slope	$\Gamma_1(i-i^*)$	for all i
	Social Structure Curve	$\Gamma_1(i-i^*)+\widetilde{\alpha}_i$	for all i
	Social Structure Surface	$\Gamma_1(i-i^*)+\widetilde{\alpha}_i+\widetilde{\beta}_j$	for combinations of i, j
	Local Social Structure Curves	$\Gamma_1(i-i^*)+\widetilde{\alpha}_i+\widetilde{\beta}_{[i+k-I]}$	for all i in each mobility group k
Dynamic Inequality	Social Mobility Slope	$\Gamma_2(k-k^*)$	for all k
	Social Mobility Curve	$\Gamma_2(k-k^*)+\widetilde{\gamma}_k$	for all k
	Social Mobility Surface	$\Gamma_2(k-k^*)+\widetilde{\gamma}_k+\widetilde{\beta}_j$	for combinations of k, j
	Local Social Mobility Curves	$\Gamma_2(k-k^*)+\widetilde{\gamma}_k+\widetilde{\beta}_{[k+i-I]}$	for all k in each origin group i
Structural & Dynamic Inequalities	Adjusted Marginal Destination Slope	$\Big(\Gamma_1\omega_{(i,j)}+\Gamma_2\omega_{(k,j)}\Big)(j-j^*)$	for all j
	Adjusted Marginal Destination Curve	$\Big(\Gamma_1\omega_{(i,j)}+\Gamma_2\omega_{(k,j)}\Big)(j-j^*)+\widetilde{\beta}_j$	for all j
	Overall Comparative Mobility Curve	$\phi_i+\Gamma_2(k-k^*)+\widetilde{\gamma}_k$	for all k in each origin group i
	Adjusted Comparative Mobility Curve	$\phi_i+\Gamma_2(k-k^*)+\widetilde{\gamma}_k+\widetilde{\beta}_{[i+k-I]}$	for all k in each origin group i
	Unadjusted Comparative Mobility Curve	$\phi_i+\Gamma_2(k-k^*)+\widetilde{\gamma}_k+\widetilde{\beta}_{[i+k-I]}+\eta_{i[i+k-I]k}$	for all k in each origin group i

Notes: $\Gamma_1=\alpha+\beta$ and $\Gamma_2=\gamma+\beta$. The quantity ϕ_i is equal to $\Gamma_1(i-i^*)+\widetilde{\alpha}_i$, which is a single value for a given origin group i.

Adjusted Marginal Destination Curve

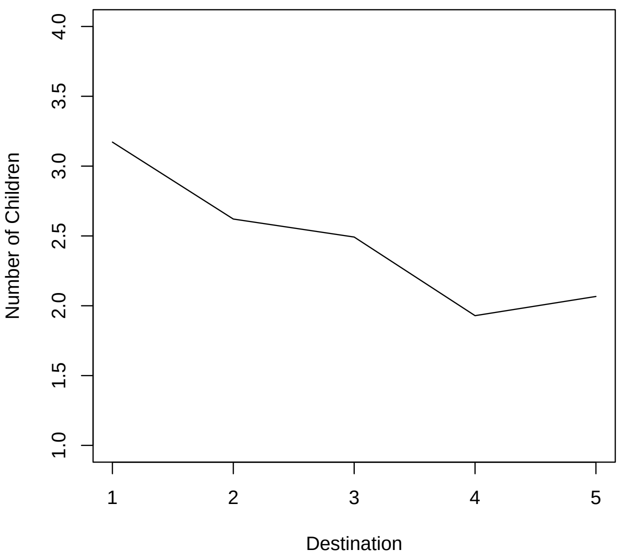
To illustrate the analytic use of just one additional expression from Table F.1, we now discuss the *adjusted marginal destination curve*. So far we have focused on examining the data through the lens of the SDI model, which is a general model of the form $Y=f(O,M)+\epsilon$. However, as we outlined in earlier sections, it may be useful in some circumstances to also examine the data using a marginal class destination model, which has the general form of $Y=f(D)+\epsilon$, where again, without loss of generality, ϵ is a normally distributed error term with a mean of zero. A particularly useful summary is the adjusted marginal destination curve, which is equal to:

$$\beta_M(j-j^*)+\widetilde{\beta}_j=\Big(\Gamma_1\omega_{(i,j)}+\Gamma_2\omega_{(k,j)}\Big)(j-j^*)+\widetilde{\beta}_j \text{ for } j=1,\ldots,J, \tag{F.1}$$

where $\Gamma_1 = \alpha + \beta$; $\Gamma_2 = \gamma + \beta$; $\omega_{(i,j)}$ is the relationship between the origin linear component and the destination linear component conditional on the origin, destination, and mobility nonlinearities; and, lastly, $\omega_{(k,j)}$ is the relationship between the mobility linear component and the destination linear component again conditional on the origin, destination, and mobility nonlinearities. This curve is equivalent to a simple class destination model (see Equation E.6 in Online Appendix E), but we have adjusted for the origin, destination, and mobility nonlinearities. Failing to adjust for the origin and mobility nonlinearities will introduce bias into the estimated overall class destination gap. However, a more practical reason for adjusting for the origin and mobility nonlinearities is that we can decompose the overall (linear) class destination gap into structural and dynamic components. This allows us to answer crucial questions regarding the extent to which cross-destination differences are attributable to differences in the social structure versus social mobility.

The $\widehat{\Gamma}_1\widehat{\omega}_{(i,j)}$ term in Equation F.1 gives the contribution of social structure to the adjusted class destination slope, while $\widehat{\Gamma}_2\widehat{\omega}_{(k,j)}$ gives the contribution of social mobility. In general, social structure will contribute more to the class destination slope the greater the degree of correlation between class origin and destination, as well as the greater the relationship between the ST slope and the outcome. Similarly, in general, social mobility will contribute more to the class destination slope the greater the degree of correlation between class mobility and destination, as well as the stronger the relationship between the SM slope and the outcome. If social mobility does not vary, destination gaps reflect only social structure; if there is no structural inequality, they reflect only social mobility. In other words, the SDI model can be used to decompose any marginal class destination gap, i.e., the kind of social class gaps that are arguably the most common estimand in social stratification research, into distinct structural- versus mobility-based components.

Figure F.1: Adjusted Marginal Destination Curve



Notes: Panel shows the adjusted marginal class destination curve, which is estimated conditional on the origin and mobility nonlinearities. The curve is a function of $\beta_M(j - j^*) + \widetilde{\beta}_j$ for all class destination groups j. Data are based on Sobel (1981).

Figure F.1 shows the adjusted marginal class destination curve for the fertility data. As can be seen, there is a general observed decline in fertility as one compares lower versus higher class destinations. The underlying adjusted marginal destination slope is $\widehat{\beta}_M = -0.290$, indicating a negative relationship between fertility and class destination. Using Equation F.1, we can decompose this overall slope into structural and social mobility components:

$$\widehat{\beta}_M = \left(\widehat{\Gamma}_1\widehat{\omega}_{(i,j)} + \widehat{\Gamma}_2\widehat{\omega}_{(k,j)}\right) = (-0.317)(0.740) + (-0.213)(0.260) = (-0.235) + (-0.055), \tag{F.2}$$

the sum of which equals adjusted marginal destination slope, or -0.290. In this case, most of the class destination gap is a function of structural differences, reflecting both the relatively large ST slope as well as the strong relationship between the class destination linear component and class destination origin linear component.

Online Appendix G: Supplemental Tables and Figures

Table G.1: Bounding Formulas for Slopes

Origin Bounds:	$\alpha_{\min} \leq \alpha \leq \alpha_{\max}$
	$\Gamma_1 - \alpha_{\max} \leq \beta \leq \Gamma_1 - \alpha_{\min}$
	$(\Gamma_2 - \Gamma_1) + \alpha_{\min} \leq \gamma \leq (\Gamma_2 - \Gamma_1) + \alpha_{\max}$
Destination Bounds:	$\Gamma_1 - \beta_{\max} \leq \alpha \leq \Gamma_1 - \beta_{\min}$
	$\beta_{\min} \leq \beta \leq \beta_{\max}$
	$\Gamma_2 - \beta_{\max} \leq \gamma \leq \Gamma_2 - \beta_{\min}$
Mobility Bounds:	$(\Gamma_1 - \Gamma_2) + \gamma_{\min} \leq \alpha \leq (\Gamma_1 - \Gamma_2) + \gamma_{\max}$
	$\Gamma_2 - \gamma_{\max} \leq \beta \leq \Gamma_2 - \gamma_{\min}$
	$\gamma_{\min} \leq \gamma \leq \gamma_{\max}$

Notes: Origin, destination, and mobility slopes are α, β, and γ, respectively, with $(.)_{\min}$ and $(.)_{\max}$ denoting minimum and maximum values of the bounds. We denote $\Gamma_1 = \alpha + \beta$, $\Gamma_2 = \beta + \gamma$, $\Gamma_1 - \Gamma_2 = \alpha - \gamma$, and $\Gamma_2 - \Gamma_1 = \gamma - \alpha$.

Table G.2: Bounds Given by Setting the Sign of One or Two Slopes

Sign of One Slope	Origin	Destination	Mobility
If $\alpha \geq 0$ then:	$0 \leq \alpha < +\infty$	$-\infty < \beta \leq \Gamma_1$	$(\Gamma_2 - \Gamma_1) \leq \gamma < +\infty$
If $\alpha \leq 0$ then:	$-\infty < \alpha \leq 0$	$\Gamma_1 \leq \beta < +\infty$	$-\infty < \gamma \leq (\Gamma_2 - \Gamma_1)$
If $\beta \geq 0$ then:	$-\infty < \alpha \leq \Gamma_1$	$0 \leq \beta < +\infty$	$-\infty < \gamma \leq \Gamma_2$
If $\beta \leq 0$ then:	$\Gamma_1 \leq \alpha < +\infty$	$-\infty < \beta \leq 0$	$\Gamma_2 \leq \gamma < +\infty$
If $\gamma \geq 0$ then:	$(\Gamma_1 - \Gamma_2) \leq \alpha < +\infty$	$-\infty < \beta \leq \Gamma_2$	$0 \leq \gamma < +\infty$
If $\gamma \leq 0$ then:	$-\infty < \alpha \leq (\Gamma_1 - \Gamma_2)$	$\Gamma_2 \leq \beta < +\infty$	$-\infty < \gamma \leq 0$

Sign of Two Slopes	Origin	Destination	Mobility
If $\alpha \geq 0$ and $\beta \geq 0$ then:	$0 \leq \alpha \leq \Gamma_1$	$0 \leq \beta \leq \Gamma_1$	$(\Gamma_2 - \Gamma_1) \leq \gamma \leq \Gamma_2$
If $\alpha \leq 0$ and $\beta \leq 0$ then:	$\Gamma_1 \leq \alpha \leq 0$	$\Gamma_1 \leq \beta \leq 0$	$\Gamma_2 \leq \gamma \leq (\Gamma_2 - \Gamma_1)$
If $\beta \geq 0$ and $\gamma \geq 0$ then:	$(\Gamma_1 - \Gamma_2) \leq \alpha \leq \Gamma_1$	$0 \leq \beta \leq \Gamma_2$	$0 \leq \gamma \leq \Gamma_2$
If $\beta \leq 0$ and $\gamma \leq 0$ then:	$\Gamma_1 \leq \alpha \leq (\Gamma_1 - \Gamma_2)$	$\Gamma_2 \leq \beta \leq 0$	$\Gamma_2 \leq \gamma \leq 0$
If $\alpha \geq 0$ and $\gamma \leq 0$ then:	$0 \leq \alpha \leq (\Gamma_1 - \Gamma_2)$	$\Gamma_2 \leq \beta \leq \Gamma_1$	$(\Gamma_2 - \Gamma_1) \leq \gamma \leq 0$
If $\alpha \leq 0$ and $\gamma \geq 0$ then:	$(\Gamma_1 - \Gamma_2) \leq \alpha \leq 0$	$\Gamma_1 \leq \beta \leq \Gamma_2$	$0 \leq \gamma \leq (\Gamma_2 - \Gamma_1)$

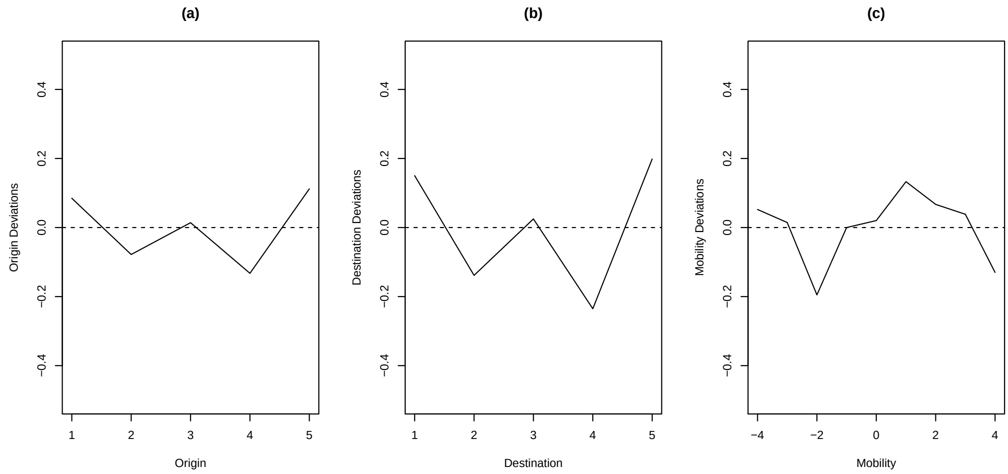
Notes: Origin, destination, and mobility slopes are α, β, and γ, respectively, with $(.)_{\min}$ and $(.)_{\max}$ denoting minimum and maximum values of the bounds. We denote $\Gamma_1 = \alpha + \beta$, $\Gamma_2 = \beta + \gamma$, $\Gamma_1 - \Gamma_2 = \alpha - \gamma$, and $\Gamma_2 - \Gamma_1 = \gamma - \alpha$.

Table G.3: Relationships among ST, SM, & Intra-Destination Slopes

Social Structure (ST) Slope:	If $\Gamma_1 = 0$,	then:	$\Gamma_2 - \Gamma_1$	$=$	Γ_2
	If $\Gamma_1 > 0$,	then:	$\Gamma_2 - \Gamma_1$	$<$	Γ_2
	If $\Gamma_1 < 0$,	then:	$\Gamma_2 - \Gamma_1$	$>$	Γ_2
Social Mobility (SM) Slope:	If $\Gamma_2 = 0$,	then:	$\Gamma_1 - \Gamma_2$	$=$	Γ_1
	If $\Gamma_2 > 0$,	then:	$\Gamma_1 - \Gamma_2$	$<$	Γ_1
	If $\Gamma_2 < 0$,	then:	$\Gamma_1 - \Gamma_2$	$>$	Γ_1
Intra-Destination Origin Slope:	If $\Gamma_1 - \Gamma_2 = 0$,	then:	Γ_1	$=$	Γ_2
	If $\Gamma_1 - \Gamma_2 > 0$,	then:	Γ_1	$>$	Γ_2
	If $\Gamma_1 - \Gamma_2 < 0$,	then:	Γ_1	$<$	Γ_2
Intra-Destination Mobility Slope:	If $\Gamma_2 - \Gamma_1 = 0$,	then:	Γ_2	$=$	Γ_1
	If $\Gamma_2 - \Gamma_1 > 0$,	then:	Γ_2	$>$	Γ_1
	If $\Gamma_2 - \Gamma_1 < 0$,	then:	Γ_2	$<$	Γ_1

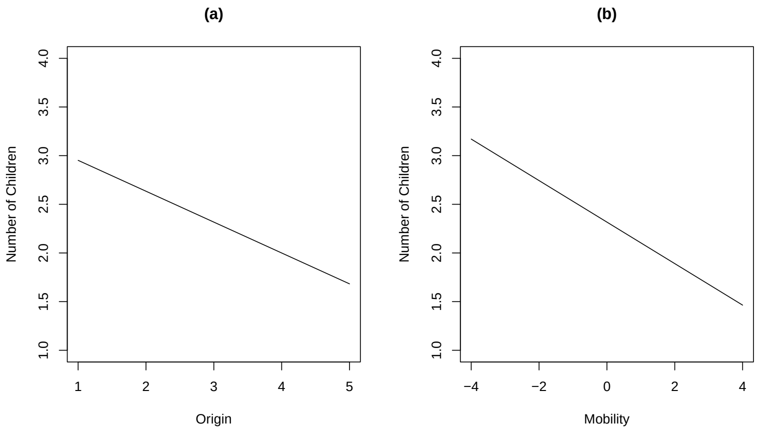
Notes: $\Gamma_1 = \alpha + \beta$ and $\Gamma_2 = \gamma + \beta$.

Figure G.1: Nonlinearities for Origin, Destination, and Mobility



Notes: Panels (a), (b), and (c) show the origin, destination, and mobility nonlinearities, which are constrained to sum to zero. Data are based on Sobel (1981).

Figure G.2: Social Structure and Social Mobility Slopes



Notes: Panel (a) shows the social structure (ST) slope, while panel (b) shows the social mobility (SM) slope. $\Gamma_1 = \alpha + \beta$ and $\Gamma_2 = \gamma + \beta$. Data are based on Sobel (1981).

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